

# Philips Technical Review

DEALING WITH TECHNICAL PROBLEMS  
RELATING TO THE PRODUCTS, PROCESSES AND INVESTIGATIONS OF  
THE PHILIPS INDUSTRIES

EDITED BY THE RESEARCH LABORATORY OF N.V. PHILIPS' GLOEILAMPENFABRIEKEN, EINDHOVEN, NETHERLANDS

## A NEW THERMIONIC CATHODE FOR HEAVY LOADS

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621.3.032.213.2:  
621.385.13.029.6

*The fact that the science of electronics — that branch of electrotechnical engineering dealing with free electrons — has become of such great importance is for the greater part due to the ease with which free electrons can be produced: a conductor or semi-conductor is heated and electrons “evaporate” from it. The thermionic cathode based upon this principle has become an integral part of innumerable types of electronic valves, a part to which little attention is now usually paid in the descriptions of the valves because in their construction the cathode does not give rise to any particular difficulties. Various modern types of valves, however, especially those for ultra short waves, have to satisfy such extreme demands that the cathode has again become an “interesting” part. A new cathode developed in the Philips Laboratories at Eindhoven (Holland), designated as the L cathode, will be able to answer these demands in many cases, due, in part, to its high maximum emission; it can yield some hundreds of amperes per sq.cm of its surface and still retain a reasonable length of life.*

### Different types of thermionic cathodes

Nowadays three types of thermionic cathodes are being widely used in electronic valves: the tungsten cathode, the thoriated tungsten cathode and the oxide-coated cathode. Table I gives a survey of a number of characteristic features of these cathodes.

The oxide-coated cathode has by far the best thermal efficiency, requiring the least heating power for a given electronic emission. Consequently this cathode is indicated for radio receiving valves, where the filament current consumption is an important point of consideration. Adverse proper-

ties of the oxide-coated cathode, such as its susceptibility to „poisoning” (reducing the emission) through traces of oxygen or other gases in the valve, and its evaporation of barium and strontium, which may cause the grids and the anode to emit electrons too, are not very objectionable in this case. For transmitting valves and X-ray tubes, however, the last-mentioned drawback is a very great objection. Moreover, for these valves and tubes the cathode must be able to stand up against the electrostatic forces of attraction of the anode, which is at a high potential, and this is not a property

Table I. Properties of some common types of cathodes.

| Type                    | Maximum useful thermionic emission in A/cm <sup>2</sup> | Maximum useful thermal efficiency in A/W | Susceptibility to poisoning | Proof against high tension | Proof against high-velocity gas ions |
|-------------------------|---------------------------------------------------------|------------------------------------------|-----------------------------|----------------------------|--------------------------------------|
| Tungsten                | 1                                                       | 0.006                                    | slight                      | good                       | good                                 |
| Thoriated tungsten      | 2                                                       | 0.070                                    | great                       | good                       | poor                                 |
| Oxide-coated cathode *) | D.C.                                                    | 0.5                                      | great                       | poor                       | during a short time good             |
|                         | pulse emission                                          | 50                                       | 20                          | great                      |                                      |
|                         |                                                         |                                          | great                       | fair                       |                                      |

\*) BaCO<sub>3</sub> and SrCO<sub>3</sub> applied in a ratio of about 1:1 by wt.

possessed by the normal oxide-coated cathode. Since in these cases the thermal efficiency of the cathode is of less importance, tungsten or thoriated tungsten cathodes can quite well be used.

Whereas for the valves and tubes mentioned, and also for cathode-ray tubes, high-tension valves, iconoscopes, etc., one of these three types of cathodes has therefore always provided a more or less satisfactory solution, this has not always been found possible for modern varieties of electronic valves such as are now used for generating or amplifying ultra short waves, because in some cases entirely new combinations of properties are demanded. In the case of magnetrons for radar installations for instance the cathode is required to yield momentarily an emission with current densities <sup>1)</sup> of some tens of amperes per sq.cm, whilst at the same time it must be proof against high tensions and also against a bombardment by accelerated electrons returning to the cathode. To a certain extent this has been provided against by reinforcing the oxide coating with metal, mostly nickel, applied in the form of a gauze or of a coarse powder sintered onto the support <sup>2)</sup>. Another, fundamental, difficulty that arises with valves for ultra short waves concerns their degassing: such valves usually contain relatively large parts made of copper, which may not be heated to such a high temperature as is desired for driving out the last traces of gas. Consequently the activation of an oxide-coated cathode in such a valve is hampered on account of the gases thereby released from the copper, poisoning this type of cathode and more or less permanently damaging it. On the other hand cathodes of tungsten or thoriated tungsten are as a rule unsuitable for use in ultra-short-wave valves because in order to obtain a reasonably high power the cathodes must have a very high specific emission (sometimes in the form of pulses) on account of the very small dimensions prescribed by the wavelength.

Having regard to the requirements for a particular kind of valve, Philips Laboratories at Eindhoven have recently developed a new type of cathode combining great mechanical strength with favourable thermionic properties and with high resistivity against impurities. From these and some other good properties it is to be expected that this cathode, which is to be designated as the L cathode, will be excellently suitable for some modern

types of valves and that the limitations hitherto set by the cathode in the construction of these valves will now be removed. Experience with several types of valves has in fact already confirmed this expectation.

The construction of this L cathode will now be described, and this will be followed by an account of its main properties and an explanation of the mechanism of its emission.

### Construction of the L cathode

Figs 1a and 1b represent two forms of the L cathode in cross section. Cathodes of the design shown in fig. 1a have a cylindrical emitting surface and can be used for instance in magnetrons, whilst those of the design in fig. 1b have a flat, circular, emitting surface and are suitable for

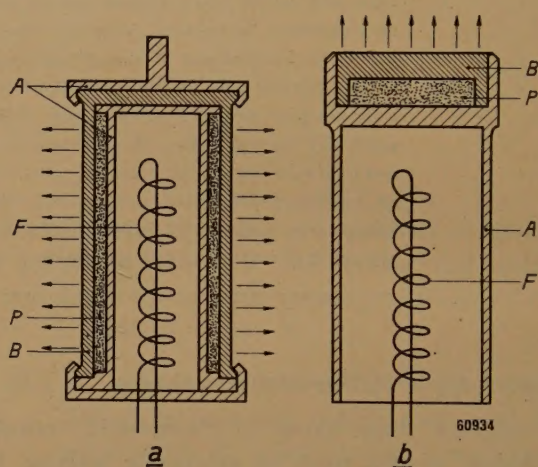


Fig. 1. Cross section of two basic forms of the L cathode. a) With cylindrical emitting surface. b) With flat, circular, emitting surface. A = wall of molybdenum, B = wall of porous tungsten, P = tablet of barium-strontium carbonate, F = filament.

velocity-modulation and reflex-oscillator valves ("klystrons"), disc-seal diodes and triodes, and also, for example, for cathode ray tubes and iconoscopes. For the sake of simplicity the further description will be confined to the form of cathode depicted in fig. 1b.

The cathode consists of two chambers shaped out of one piece of molybdenum. The lower chamber is open at one end and contains an insulated filament for indirect heating of the cathode. The upper chamber is closed with a cap of porous tungsten, underneath which is a tablet of barium-strontium-carbonate (in the proportions of about 1:1 by weight); the only connection with the outside from this upper chamber is via the pores of the tungsten. Supporting rods can be welded to the bottom end of the cathode for fixing it in the right place.

<sup>1)</sup> The emission need not be purely thermionic, but can be obtained partly as secondary emission; this will be dealt with farther on.

<sup>2)</sup> J. B. Fisk, H. D. Hagstrum and P. L. Hartman, Bell Syst. Techn. J. 25, 167-348, 1946.

The cathode of the design according to fig. 1a is likewise divided into two chambers, that containing the barium-strontium-carbonate being connected with the outside only via a jacket of porous tungsten.

All sorts of variations of these two basic forms of the construction of the L cathode are possible, some of which are illustrated in fig. 2.

This temperature-dependency is graphically represented in fig. 3 for the L cathode and for the three types of cathodes given in table I, by plotting the logarithm of the saturation emission as a function of the temperature. The saturation emission is about the maximum emission to be obtained from a cathode at a given temperature.

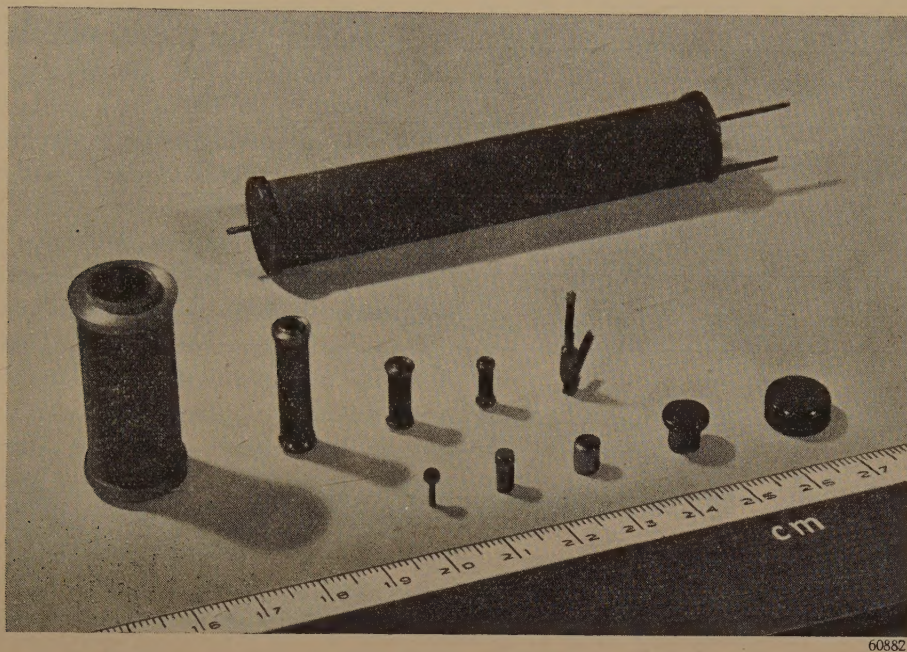


Fig. 2. Various forms of execution of the L cathode, for different tubes.

The cap (fig. 1b) or the cylinder (fig. 1a) of porous tungsten is made by compressing tungsten powder under high pressure and then sintering it at high temperatures.

After it has been mounted in a valve the cathode is activated by heating it in vacuo first to a temperature of about 1100 °C, whereby the barium-strontium carbonate gives off carbon dioxide and is converted into barium-strontium oxide, and then, after the carbon dioxide escaping through the porous tungsten wall has been pumped off, to a higher temperature. During this further heating the cathode begins to emit electrons thermally and sooner or later, depending on the temperature, the emission reaches an almost constant value. The cathode is then ready for use.

#### Thermionic emission of the L cathode

What is primarily of interest when comparing different cathodes is the temperature-dependency of the emission (by emission is to be understood here the density of the electron current emitted, in A/cm<sup>2</sup>).

As regards the temperature required to obtain a given saturation emission, fig. 3 shows that the L cathode occupies a place between the oxide-coated cathode and the thoriated tungsten cathode. In this connection it is to be noted that when an oxide-coated cathode is used continuously no more than about one hundredth part of the saturation emission may be taken from it, as otherwise the emitting layer would suffer damage. The other types of cathodes can operate continuously with an emission close to the saturation value. Consequently, for a certain desired emission the working temperature of the oxide-coated cathode cannot be chosen quite so low as might be concluded from fig. 3.

Turning now to the question as to what is the maximum emission that can be obtained, i.e. the extreme point to which the curves in fig. 3 have been drawn, it is seen that with the L cathode we have got much farther than with any other type of cathode. The supremacy of the L cathode is even more impressive when it is realized that the termination of the curves is not due to the same

causes in all cases. The curves for the three types of cathodes previously mentioned do not extend any farther because at higher temperatures the cathodes are very soon destroyed; the extreme end of the curve for the oxide-coated cathode, extending to about 120 A/cm<sup>2</sup>, can even only be reached with pulse emission. In the case of the

generously loaded cathode area takes part in the emission. We used for this purpose a flat cathode, with an emitting surface of say 6 mm<sup>2</sup>, and a flat anode in parallel. The distance between cathode and anode was made very small, so that for an emission of 40 A/cm<sup>2</sup> (the heaviest D.C. load of the L cathode so far reached) the anode voltage required in view

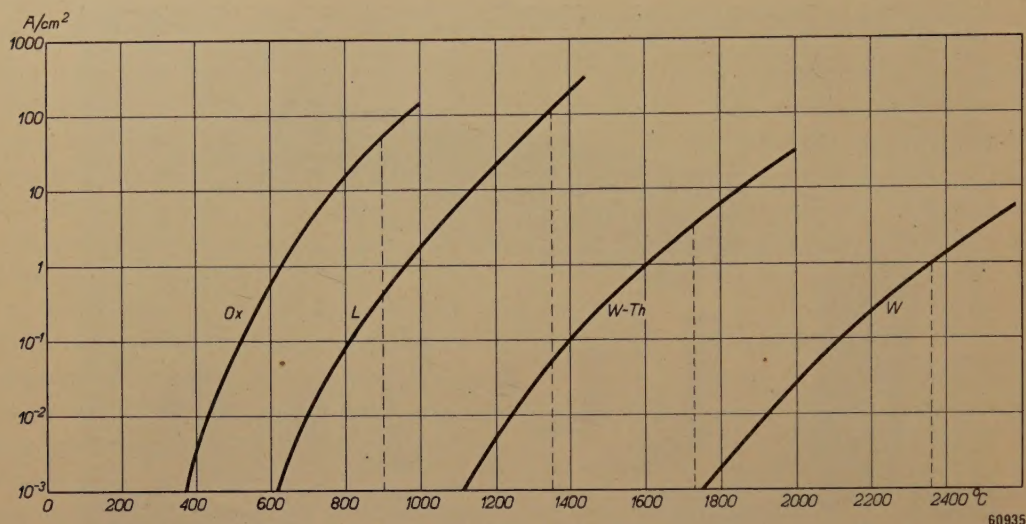


Fig. 3. Saturation emission  $J_s$  (in amperes/cm<sup>2</sup>) as function of the temperature (in °C) for the L cathode and three other types of cathodes. (The curves for these three types and the data for the corresponding curves in figs 4 and 5 and for table II have been taken from: G. Hermann and S. Wagener, *Die Oxydkathode*, publ. J. A. Barth, Leipzig 1944, Vol. II, pp. 79-80). Along the abscissa, as also in figs 4 and 5, the true temperature has been plotted (not the "black" temperature, as is often taken). For each cathode a vertical dotted line has been drawn to indicate approximately the maximum temperature at which the cathode can be used if it is desired to have a life of at least some hundreds of hours. In the case of the curve for the oxide-coated cathode it is to be borne in mind that these emission values can only be used under pulse conditions; for D. C. emission only about one-hundredth part can be used. The curves end at the temperature where the respective cathode would very soon break down; the end point for the L cathode, however, is determined rather by the difficulties encountered in measuring, since in this case exceptionally high emissions are reached. It is this high maximum emission (together with other properties which are dealt with in the text but cannot be expressed in the emission curve) that typifies the L cathode.

L cathode, however, the extreme point of the curve, lying at about 300 A/cm<sup>2</sup>, which in principle is to be reached equally well with continuous as with pulse emission, is in point of fact not determined by the cathode itself but by difficulties in measuring. At a pulse emission of about 300 A/cm<sup>2</sup> in the measuring tubes used, a gas discharge took place, presumably due to the formation of gas from the anode material as a result of the sudden heating: during a pulse the anode dissipates several megawatts per sq.cm. When measuring with continuous emission this anode dissipation gives rise to still greater difficulties. Naturally the measurements have to be taken with a system of electrodes in which an accurately known and homo-

of the counteracting space charge amounted to only 400 volts. With that emission this still means a dissipation of 1 kW, on a small anode surface; even with water-cooling it was not easy to carry off such a power.

It is quite possible that in practice better solutions can be found for these difficulties of dissipation than were possible with the tubes built for carrying out the measurements, and that still higher emissions can then be obtained from the L cathode. Anyhow the foregoing shows that, thanks to the L cathode, the emission attainable need no longer constitute an obstacle for the development of new valves.

In addition to the maximum emission and the

temperature-dependency of the emission there is also the thermal efficiency of the cathode to be considered, by which is to be understood the maximum thermionic current emitted in amperes per watt heating power. The heating power required is in the first instance determined only by the thermal losses due to radiation and conduction at the temperature at which the cathode yields the desired electron emission. Therefore the lower the temperatures of the emission curve in fig. 3, and the more the cathode differs from the black body — that is to say, the less the radiation from the cathode at a given temperature — the smaller will be the heating current required. As far as the thermal losses through conduction (via the supporting rods, etc.) are concerned, these are not usually taken into account when comparing different types of cathodes, because these losses depend to a high degree upon the construction of the valve. Also the radiation from parts of the cathode not contributing towards the thermionic emission is disregarded.

The so defined theoretical thermal efficiencies, where only the temperature-dependency of the emission and the radiation properties of the cathode surface play a part, have been plotted in fig. 4 as a function of the saturation emission, again both for the L cathode and for the three other types of cathodes. From this diagram it can be seen that the theoretical efficiency of the L cathode is greater than that of the tungsten and thoriated tungsten cathodes but less than that of a normal oxide-coated cathode. It is not surprising that the latter should be the case, considering that the temperature of the L cathode is higher than that of the oxide-coated cathode, with equal saturation emission, whilst moreover the porous metal surface of the L cathode is a good radiator of heat.

From the foregoing it can already be concluded that for valves for which relatively small current densities are needed, say less than  $0.25 \text{ A/cm}^2$ , and for which the smallest possible heating power is required (as is the case for normal radio receiving valves), the oxide-coated cathode is to be given preference over the L cathode<sup>3</sup>). In cases where higher emission currents are required, however, it is necessary to consider the thermal efficiency somewhat more closely before drawing any conclusions.

Anticipating the discussion of the emission mechanism it can be stated that there is a relation

<sup>3</sup>) This still holds when it is taken into account that only a fraction of the saturation emission of the oxide-coated cathode may be used for D.C. loads.

between the thermionic properties of the various types of cathodes and their work function  $\varphi$ ; before an electron can leave the cathode and enter the vacuum it has to overcome at the surface a potential difference  $\varphi$ , the value of which is characteristic for the type of cathode. At room temperature only few electrons possess the necessary energy  $e\varphi$  ( $e$  = the charge of the electron) to overcome this potential difference. This number rapidly increases as the temperature rises and a measurable

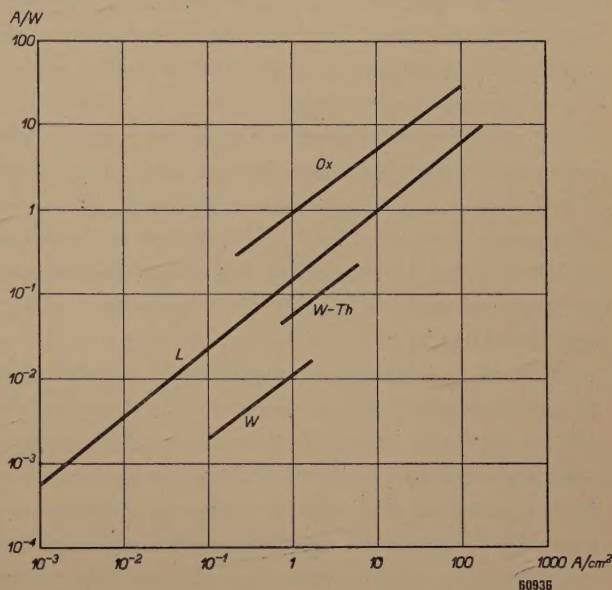


Fig. 4. Theoretical thermal efficiency (in amperes per watt heating power) as function of the saturation emission  $J_s$  for the L cathode and three other types of cathodes.

quantity of electrons are emitted. Of course the same number of electrons are restored to the cathode from the supply lead. Since the electrons possessing most energy are selected for the emission, whilst this is not the case with the electrons fed to the cathode, the result is that in the emission the cathode loses energy, or in other words it cools down. Where a large number of electrons are emitted this effect is quite noticeable, as may be illustrated by an example: when working with a D.C. emission of  $40 \text{ A/cm}^2$  and taking for  $\varphi$  a value of  $1.8 \text{ V}$  (a normal value for the L cathode) the electrons emitted carry  $40 \times 1.8 = 72 \text{ watts/cm}^2$  out of the cathode (actually a little more than this, because the electrons leave the cathode at a fairly considerable velocity). The heating power must therefore be increased by a like amount in order to keep the cathode surface at the temperature required for the emission. Without emission, thus for covering the radiation losses only (theoretical efficiency), a current of  $20 \text{ W/cm}^2$  would be sufficient for this temperature.

In the case of the oxide-coated cathode the cooling effect is about  $1\frac{1}{2}$  times smaller, since the work function  $\varphi = 1.0$  to  $1.5$  V. At the same time, however, with this cathode another effect arises, which acts in the opposite direction. The emitting oxide coating is a semi-conductor, thus having a fairly high resistance  $R$  for the electron current (anode current) that has to traverse it before it can be emitted. Consequently in the oxide coating, owing to the Joule effect, the emitted current  $i$  develops a certain amount of heat  $i^2 \times R$ , which with high currents, such as are drawn from the oxide-coated cathode in the case of pulse emission, assumes very large values <sup>4</sup>). The cooling effect, which increases only in proportion to  $i$ , may be entirely overshadowed by the heat due to the Joule effect and in some cases there may be a rise in temperature of  $100^\circ\text{C}$ . (With an emission of about  $1\text{ A/cm}^2$ , thus near the limit for D.C. emission of the oxide-coated cathode, these two effects are about equal.)

It would be wrong to regard this Joule effect as another favourable factor for the thermal efficiency of the oxide-coated cathode. Although, as a consequence of the Joule effect, for the temperature desired a still smaller heating current can (or rather, must) be applied to the oxide-coated cathode, this only means that part of the power for the heating is drawn from the source of supply for the anode current instead of from the source of the cathode current, whilst the total power remains unchanged. Thus no advantage whatever is derived from the Joule effect and in fact it has even a disadvantage: the part of the energy contributed by the anode-current source depends upon the resistance of the oxide coating and may therefore vary slightly for different valves, whilst it also varies with the load of the valve, so that it is less easy to control the cathode temperature than when the cathode is heated exclusively by the filament current.

Looked at in this light, the absence of this Joule effect is to be regarded as an important advantage of the L cathode. It is true that, as a result of the cooling effect, which is not compensated in this case, the thermal efficiency reached in practice is even lower than the theoretical efficiency, but in those cases where a large emission is required, and thus the cooling effect becomes noticeable, the efficiency of the cathode will not usually be

a factor of any great importance. Difficulties with dissipation do not arise from the additional heating current required, because this only leads to a very small percentual increase of the anode dissipation.

### Other properties of the L cathode

In outward appearance the most striking feature of the L cathode is its smooth, metallic, emitting surface. This gives the cathode great mechanical strength, it is not liable to damage while being mounted, and there is no risk of particles peeling off under the influence of electrostatic forces of attraction. Thus a great drawback of the oxide-coated cathode, with its mechanically weak layer of carbonate or oxide, is avoided. Furthermore, the emitting surface of the L cathode, which is turned on a lathe or moulded, can easily be made perfectly flat and exactly to given dimensions, within tolerances of only a few microns. For valves where it is of importance that the cathode should be at a very short distance (of the order of some tens of microns) from the other electrodes in order to limit the transit times of the electrons, as is the case with disc-seal valves, this offers very good possibilities for their construction, better even than with the finest-grained oxide-coated cathodes, because these have to be handled with the utmost care and are therefore difficult to mount with the necessary precision.

It is also to be pointed out that for the kind of ultra-short-wave valves just mentioned the high emission that can be obtained continuously with the L cathode yields a two-fold advantage. Not only can the cathode surface be made very small, thus giving extremely small valve capacitances — which is of importance for a good quality of the oscillatory circuits at very high frequencies — but a great density of current requires a high control voltage, which further shortens the transit times of the electrons and thus reduces the “transit-time damping”. Also the fact that the L cathode has no cross resistance worth mentioning — in contrast to the emitting layer of the oxide-coated cathode, see above <sup>5</sup>) — helps to reduce the damping of the circuits, at least at wavelengths of the order of  $1\text{ m}$  (at shorter wavelengths the effect of the resistance of the oxide-coated cathode again decreases owing to the capacitive component in the impedance of the oxide coating shunting the resistive component).

<sup>4</sup>) This generation of heat, as also, inter alia, phenomena of electrolysis of the coating due to the passage of the current, account for the difference between the permissible D.C. and pulse emission of the oxide-coated cathode; cf table I.

<sup>5</sup>) Cf also R. Loosjes and H. J. Vink, Conduction processes in the oxide-coated cathode, Philips Techn. Rev. 11, 271-278, 1949/1950 (No. 9).

As was to be expected, the L cathode has proved to be fully capable of withstanding the heavy electron bombardment taking place in magnetrons and also in reflex valves<sup>6)</sup>, in which respect it compares very favourably with the oxide-coated cathode. In two similar reflex valves working on a wavelength of 10 cm and with a continuous cathode load of up to about 2 A/cm<sup>2</sup> an oxide-coated cathode lasted no longer than about 100 hours, whereas an L cathode continued to work for more than 1000 hours.

When the electrons returning to the cathode in magnetrons and reflex valves have a sufficiently high velocity, then the secondary emission from the L cathode is such that the ratio of the number of emerging secondary electrons to the number of incident primary electrons is appreciably greater than 1, as is the case with most metals. In so far as it may be of importance to effect a saving in heating power or to increase the thermionic emission beyond what is thermionically possible with the L cathode, advantage can be taken of this property (which in the case of the oxide-coated cathode for magnetrons is practically a *conditio sine qua non*).

As regards the behaviour of the L cathode in a magnetron, provisional tests show that good results are to be expected also with this type of valve. Other investigators in the Philips laboratories will deal with this application of the L cathode on a later occasion.

The drawback of the oxide-coated cathode that small quantities of barium and strontium evaporate from it applies likewise to the L cathode, as will be shown at the end of this article. The L cathode also tends to be poisoned to a certain extent by oxygen or oxygen compounds (e.g. carbon monoxide) present in the valve, as is also the case with the oxide-coated cathode, but when the normal vacuum has been restored then the thermionic emission of the L cathode recovers more quickly and more easily than that of the oxide-coated cathode. This gives the L cathode the advantage in valves that are difficult to degas. Such applies all the more in cases where the cathode is exposed to a bombardment by high-velocity gas ions and to sparking. After a short bombardment the oxide-coated cathode recovers, but when the bombardment lasts for any length of time the whole of the emitting coating may be lost through atomization, whilst sparking makes

holes in the coating. The L cathode, on the other hand, can withstand both sparking and a lengthy bombardment by gas ions. Although the thermionic emission is greatly reduced during the bombardment, as soon as the cause of the bombardment has been removed the emission recovers and it is found that no permanent damage has been suffered. Clear proof of this has been obtained for instance with certain types of velocity-modulated valves, in which, notwithstanding the fact that the load was not very high (0.8 A/cm<sup>2</sup>), oxide-coated cathodes broke down after about 25 hours, whereas an L cathode lasts some thousands of hours.

The high degree of reliability resulting from these properties is of particular importance for the manufacture of very expensive valves and tubes (e.g. iconoscopes and super-iconoscopes). Even if these can quite well be made with an oxide-coated cathode, an L cathode is likely to be preferred in order to minimize the risk of failure.

As the last and highly important feature, the life of the cathode under normal working conditions may be discussed. Whereas the life of the oxide-coated cathode depends not only upon the temperature at which it is used but also upon the emission current taken from it, the latter factor does not play a part in the case of the L cathode. As might be expected, the higher the working temperature the shorter is the life of this cathode, but the temperature range within which the L cathode can be worked with a useful emission and a reasonable life is very extensive, much more so than in the case of the oxide-coated cathode, viz. 900 to 1350 °C as compared with 700 to 900 °C. To give some further data: at 1000 to 1100 °C the life of the L cathode is some thousands of hours, at 1250 °C some hundreds of hours, and even at 1350 °C it is still some tens of hours. For a proper appreciation of these figures it should be borne in mind that at the temperatures mentioned the saturation emission amounts respectively to 3, 100 and 250 A/cm<sup>2</sup>. Such performances are not equalled by any other type of cathode.

#### The mechanism of the emission

From the fact that both in the case of the L cathode and in that of the oxide-coated cathode a mixture of barium and strontium carbonates is employed, from which the carbon dioxide is driven out, it might be thought that the L cathode could be regarded as being merely a variation of the oxide-coated cathode. A closer investigation into the mechanism of the emission, which will

<sup>6)</sup> See, e.g., F. Coeterier, The multireflection tube, a new oscillator for very short waves, Philips Techn. Rev. 8, 257-266, 1946.

now be dealt with, shows however that this is not the case but that the L cathode represents a new type. First we shall briefly review the laws which any material emitting electrons obeys.

The temperature-dependency of thermionic emission (i.e. the function represented in fig. 3) can be generally described by Richardson's formula:

$$J_s = AT^2 \exp(-e\varphi_0/kT),$$

where  $J_s$  represents the saturation emission in A/cm<sup>2</sup>,  $T$  the absolute temperature,  $\varphi_0$  the work function in volts at absolute zero,  $e$  the charge of

the behaviour of that type by a single figure. It is then obvious that also differences in the mechanism of the emission will find expression in differences in the value of  $\varphi_0$ .

In the case of the tungsten cathode the emission takes place from the surface of the pure metal, the  $\varphi_0$  value of which reaches the very high figure of 4.5 V. The thoriated tungsten cathode has a monoatomic layer of thorium on the surface of the tungsten, and such monoatomic layers greatly reduce the high work function of tungsten: the value of  $\varphi_0$  for the thoriated tungsten cathode is

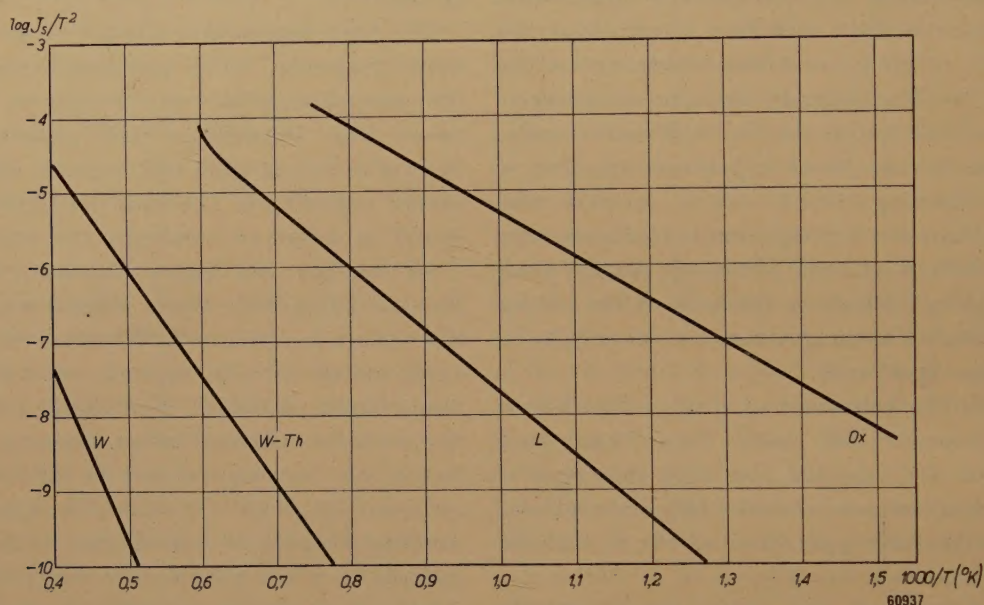


Fig. 5. Richardson lines for the L cathode and three other types of cathodes.

the electron =  $1.60 \times 10^{-19}$  coulombs,  $k$  Boltzmann's constant =  $1.38 \times 10^{-23}$  joule/degree, and  $A$  a constant of the emitting surface expressed in A/cm<sup>2</sup>/degree<sup>2</sup>.

Since in the temperature range occurring in practice in all cases  $e\varphi_0 \gg kT$ , it is clear that the variation of  $J_s$  as a function of  $T$  is determined almost entirely by the exponential function. This implies (1) that  $J_s$  varies greatly with the temperature, the more so the larger the value of  $\varphi_0$ , and (2) that for different materials, but with equal temperature, the value of  $J_s$  will be determined by the magnitude of  $\varphi_0$ , such that the smaller the value of  $\varphi_0$  the greater will be the value of  $J_s$  (the values of  $A$  for various materials do not differ so much as to have any appreciable influence affecting this conclusion).

From this it appears that the value of  $\varphi_0$  for a type of cathode can be used for indicating roughly

about 2.7 V. In the case of the oxide-coated cathode we have to do with an emitting semi-conductor, and not with an emitting metal. The emission mechanism in this case is rather complicated and we cannot enter into it here, but it results in an exceptionally low value of the work function, viz. 1.0 to 1.5 V.

Once the emission  $J_s$  of a cathode has been measured at different temperatures it is easy to determine  $\varphi_0$  by plotting  $\log(J_s/T^2)$  as a function of  $1/T$ . According to Richardson's formula the measuring points should lie on a straight line — which as a rule appears to be fairly well the case —, and the slope of the line indicates the value of  $\varphi_0$ , apart from a factor  $k/(e \log e) = 1.98 \times 10^{-4}$  V/degree. Fig. 5 shows the Richardson lines for four cathodes, the older types already mentioned and an L cathode, whilst table II gives the values of  $\varphi_0$  (and of  $A$ ) derived from such lines for a

Table II.

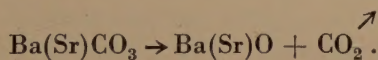
| Type of cathode      | $\varphi_0$<br>in volts | $A$<br>in A/cm <sup>2</sup> /degree <sup>2</sup> |
|----------------------|-------------------------|--------------------------------------------------|
| Tungsten             | 4.44-4.63               | 22 -210                                          |
| Thoriated tungsten   | 2.6 -2.9                | 3 - 15                                           |
| Oxide-coated cathode | 1.0 -1.5                | 0.01- 5                                          |
| L cathode            | 1.6 -2.0                | 1 - 15                                           |

series of various specimens of the four types of cathodes.

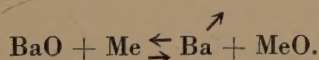
Although the extreme values for  $\varphi_0$  of the L cathode and the oxide-coated cathode closely approximate each other, the difference is still too great for these types to be regarded as being identical. To put it in other words, it is unlikely that the emission of the L cathode takes place from a layer of barium-strontium oxide. On the other hand, considering the relatively low value of  $\varphi_0$ , there cannot be any question of emission from the porous tungsten surface itself.

What, then, is the mechanism of the emission of the L cathode?

During the very first heating in vacuo the barium-strontium carbonate in the closed chamber is dissociated according to:



The carbon dioxide is pumped off. During the next heating the barium oxide is partly reduced to barium metal:



By Me is meant one of the surrounding metals (for the sake of simplicity taken to be bivalent). At the temperatures used (900 to 1350 °C) the vapour pressure of the barium metal is very high. Consequently this metal will escape from the reaction equilibrium and the reaction will move gradually to the right, notwithstanding the fact that the heat of formation of BaO, which is great compared with other metals, tends to force the reaction to the left. Thus, upon the cathode being heated, barium vapour will be formed in the closed chamber under a certain, very small, pressure that is determined by the speed of the reaction just mentioned. The same applies for the strontium <sup>7)</sup>. Further, there will be also a noticeable amount of BaO vapour, since at these temperatures also BaO has a rather considerable vapour pressure; the vapour pressure of SrO is negligible. See fig. 6.

The mixture of Ba(Sr) and BaO vapours passes out through the pores of the tungsten, and in those pores it will form a monoatomic layer on the tungsten (a multiatomic layer would evaporate again owing to the high vapour pressure, whilst the first atomic layer is retained by the forces of adsorption). Now the barium in this form of an adsorbed layer moves about over the surface <sup>8)</sup>, so that after a time the whole of the surface of the tungsten, inside and outside, will be covered with a monoatomic layer of barium, mixed with some oxygen. This layer, just like the layer of thorium in the case of the thoriated tungsten cathode, results in a considerable reduction of the work function. Thus the high thermionic emission of the L cathode is made understandable <sup>9)</sup>.

The representation of the mechanism as developed above is supported by a number of experimental facts. For instance it has been possible to prove by direct means that barium, barium oxide and

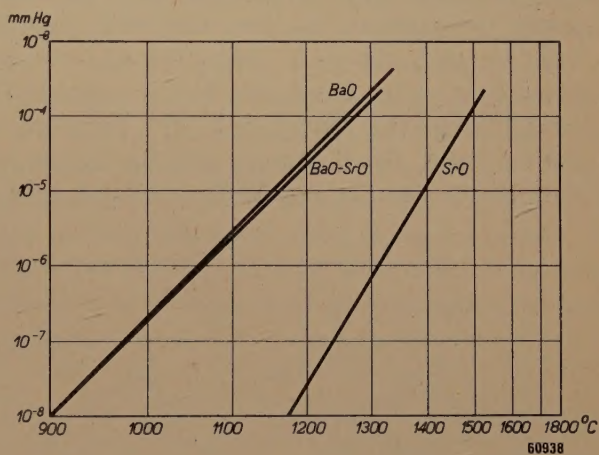


Fig. 6. Vapour pressure of BaO, BaO-SrO and SrO as function of the temperature (taken from A. Claassen and C. F. Veenemans, Z. Physik **80**, 342-351, 1933). The mixture of BaO and SrO has a molecular ratio of 42.8:57.2. The vapour of this mixture does not contain any measurable quantity of SrO.

<sup>7)</sup> The heating also reduces the strontium oxide to strontium. SrO has an even greater heat of formation than BaO, but the vapour pressure of Sr is also greater than that of Ba, so that the reaction with SrO is still noticeably directed to the right. — Exactly what part the strontium plays in the functioning of the cathode cannot yet be sufficiently explained. The strontium cannot be dispensed with since it has been proved empirically that the life of the cathode is then considerably shortened.

<sup>8)</sup> This, as well as the formation of a layer as mentioned above, has been demonstrated by, among others, J. A. Becker in Trans. Far. Soc. **28**, 148-158, 1932, and J. A. Becker and G. E. Moore in Phil. Mag. **29**, 129-139, 1940.

<sup>9)</sup> The adsorbed oxygen atoms, which in themselves obstruct the escape of electrons from the metal, have also a favourable effect in that, as negative centres, they promote a stronger binding of the adsorbed barium atoms on the surface. This favourable action predominates so long as there is not too much oxygen present. Cf. J. H. de Boer, Elektronenemission und Adsorptionserscheinungen, J. A. Barth, Leipzig 1937, page 113.

strontium evaporate from the L cathode. In the pores of the activated cathode a quantity of barium is found which corresponds fairly well to the quantity needed for the formation of a continuous monoatomic layer. An experiment carried out with a cathode without the tablet of barium-strontium carbonate showed that the cathode could be activated by evaporating upon it a monoatomic layer of barium from the outside (most probably also traces of oxygen then reach the cathode), and it was then found to have a work function  $\varphi_0 = 1.7$  volts, the same as that of a normal L cathode! When, however, a layer of barium oxide about 1 micron thick is applied to the metal surface then, at the same temperature, the emission is greater and  $\varphi_0 = 1.4$  volts, which is a noticeable difference compared with the normal L cathode.

It is therefore quite evident that the L cathode is not to be regarded as a kind of oxide-coated cathode with a thin coating of barium oxide on a support of tungsten, but rather as showing a strong resemblance to the thoriated tungsten cathode, with the part played by the thorium taken over in the L cathode by the barium. The special properties described in the foregoing as characterizing the L cathode are due to the fact that the barium in a monoatomic layer reduces

the work function of tungsten to a much greater extent than thorium does, and to the special construction which allows of the monoatomic layer being continually renewed automatically.

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**Summary.** A new thermionic cathode developed in the Philips Laboratories at Eindhoven (Holland), designated as the L cathode, has a mixture of barium and strontium oxides contained behind a wall of porous tungsten. At the working temperature of the cathode (900-1350 °C) the barium and strontium oxides are gradually reduced; owing to the fairly high vapour pressures of Ba, Sr and BaO these substances escape through the pores of the wall and form there on the tungsten surface a monoatomic layer of barium and strontium with some oxygen in between. This layer reduces the work function, which for pure tungsten is 4.5 V, to 1.6-2.0 V. This value is not quite so low as that of the oxide-coated cathode (1.0 to 1.5 V), so that the required temperature is higher and the thermal efficiency lower than in the case of the oxide-coated cathode, but owing to its construction the L cathode is much better able to withstand heavy loads (even at the higher temperature). The maximum useful emission, with a reasonable life, amounts to some hundreds of amperes per cm<sup>2</sup> (measured under pulse conditions; the maximum useful emission in the case of a D.C. load is in principle the same, but difficulties then arise in the measurement). Further the L cathode is proof against the electrostatic forces of attraction at high tensions, it easily recovers after a possible poisoning by oxygen or other gases and after a bombardment by high-velocity gas ions, and it can quite well withstand a bombardment by electrons, whereby a considerable emission of secondary electrons takes place. All these properties make the L cathode highly suitable for all types of modern valves and tubes for generating ultra short waves, and in general for those where the cathode is required to have great mechanical strength and a high degree of reliability in addition to a high emission.

# THE MANUFACTURE OF QUARTZ OSCILLATOR-PLATES

## II. CONTROL OF THE CUTTING ANGLES BY X-RAY DIFFRACTION

by W. PARRISH \*).

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*During the war X-ray diffraction, which had been used only in research in some scientific and technical laboratories, also proved its usefulness as an aid in mass production manufacturing. It was introduced into the quartz oscillator-plate industry for the measurement of the crystallographic angles of cuts from quartz crystals. The diffraction apparatus developed for this purpose makes it possible for unskilled people to measure deviations with respect to specified crystallographic angles with an accuracy of a few minutes of arc in a time of 10 to 15 seconds.*

### The tolerances of oscillator plates

As an introduction to this second article on the manufacture of oscillator plates it is desirable to consider more closely the question of tolerances in the orientation and dimensions of these plates.

It has been stated <sup>1)</sup> that the deviations in the crystallographic angles with respect to the prescribed values for a low temperature coefficient cut, a BT cut for example, may not amount to more than 10 minutes, while the thickness of a plate is restricted to a tolerance of, for instance,  $10^{-5}$  mm. Furthermore the length of the sides is often also prescribed, with a tolerance of 0.03 mm or less.

The crystallographic angles involved are illustrated in *fig. 1*, for the case of the square AT and BT plates. This figure shows how these two cuts are orientated in a quartz crystal. An edge  $X'$  of the plate must be parallel to an  $X$ -axis (electrical axis) of the quartz, thus the angle  $XX' = 0^\circ$ . The projection  $Z'$  of the  $Z$ -axis (optic axis) on the face of the plate must make an angle  $ZZ'$  with the  $Z$ -axis amounting to  $35^\circ 15'$  in the case of the AT cut and of  $-49^\circ 20'$  in that of the BT cut. Generally the tolerance for  $XX'$  is somewhat larger than for  $ZZ'$ . During the war it was in most cases  $20'$  for  $XX'$  and  $10'$  for  $ZZ'$ . In the case of the higher precision AT plates made to-day, the  $ZZ'$  angle is often held to smaller tolerances, perhaps 3 or 4 minutes.

These tolerances in the crystallographic angles are dependent upon the requirements regarding the variation of the resonance frequency as a function of the temperature. The turning point of

this curve may be displaced to an undesirable temperature region or it may disappear due to a slightly incorrect orientation of the cut. This is illustrated by *fig. 2*. The tolerance in the thickness depends upon the accuracy with which the desired resonance frequency must be realized. In the case of AT and BT plates where the desired mode of vibration involves a thickness shear, the frequency is inversely proportional to the thickness. The tolerance in the length of the sides is finally prescribed by the requirement that a coupling of the desired vibration with undesired modes of vibration of the plate must be avoided as much as possible. In the case of AT and BT plates, the high

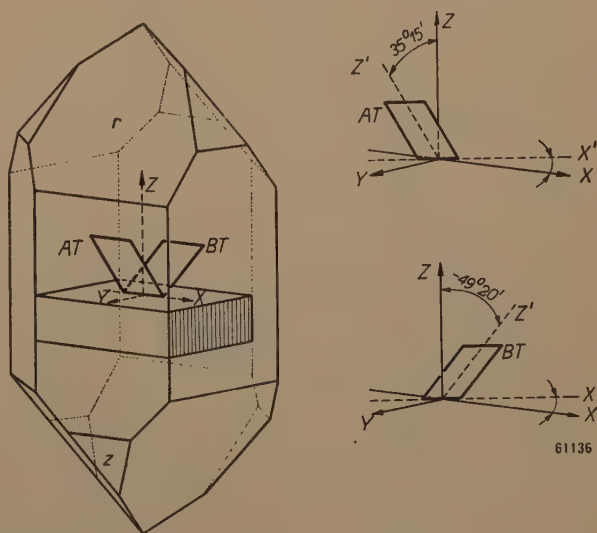


Fig. 1. Orientation of AT and BT cuts in an ideal quartz crystal (natural crystals do not often have all the regularly developed faces shown here). Both cuts, as well as most other low-temperature coefficient cuts, are perpendicular to a  $Y$ - $Z$  plane, at an angle of  $35^\circ 15'$  and  $-49^\circ 20'$ , respectively, to the  $Z$ -axis. For the BT cut the desired angle varies between  $-49^\circ 0'$  and  $-49^\circ 30'$ , dependent on the temperature range in which the oscillator plate is to be used; in the following we shall base our discussion on the first-mentioned intermediate value.

\*) Philips Laboratories, Inc., Irvington-on-Hudson, N.Y., U.S.A.

1) W. Parrish, The manufacture of quartz oscillator-plates, I. How the required cuts are obtained, Philips Techn. Rev. 11, 1949 (No. 11). This article is referred to in the following as I.

harmonics of a fundamental low frequency flexural mode about a Y-axis, which runs diagonally through the plate, often cause the most trouble.

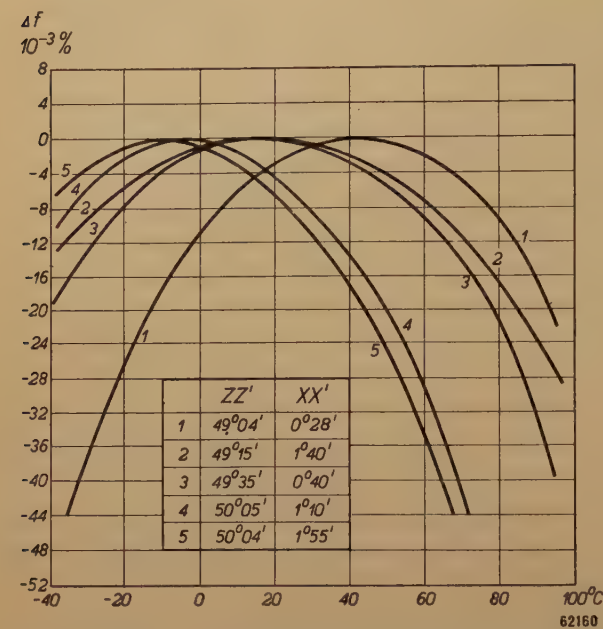


Fig. 2. Variation of resonance frequency with temperature, for a number of BT cuts with slightly differing crystallographic angles. The ordinate scale gives the deviation of the resonant frequency from the maximum value, in  $10^{-3} \%$ . The turning points (maxima) of the curves lie near 42, 18, 16, -2 and  $-10^{\circ}\text{C}$  respectively. Besides the position of the turning point, the steepness of the curve must be taken into account, in order to get a broad range of working temperatures.

The check on whether the plate has been brought within the tolerances of thickness and length is not carried out by measuring directly these dimensions but indirectly (or rather even more directly) by investigation of the behaviour of the plate in the oscillator circuit of a valve oscillator. In this way it is easy to determine the mechanical resonance frequency of the plate. The “activity” of the plate, i.e., the amplitude at which the plate vibrates, is also measured and serves as a criterion of an adequate limitation of undesired couplings: excessive couplings cause activity “dips” at some frequencies. The lapping (and etching) of the plate is continued until it satisfies the specifications as to frequency and activity. In the last article of this series we shall have an opportunity of going into this more deeply.

The proportionality constant in the relation between resonance frequency and thickness (i.e. the product of frequency times thickness) depends closely upon the orientation of the cut in the crystal. The dimensions required for the avoidance of couplings are also very much influenced by the orientation. In the case of the “indirect” empirical process of finishing the oscillator plates little attention was given to these influences. Nevertheless, the last mentioned influence is

of practical importance. Several manufacturers endeavor to minimize the coupling with undesired modes of vibration by giving the plates well defined and previously determined optimal dimensions (predimensioning). These dimensions vary with the cutting angles and hence it is necessary to maintain small tolerances for these angles to make the scheme practical. During the war predimensioning was not widely practiced in production. Activity dips over the temperature range were then one of the principal causes of rejection of plates.

The checking and correction of the cutting angles was formerly accomplished by a similar “indirect” procedure: a plate was cut from the crystal and lapped to the desired resonance frequency. The frequency was measured as a function of the temperature and from the curve obtained it was deduced as best one could by what amount the cutting angles were incorrect. It was not possible from this method to separate the corrections for ZZ' and XX', but, moreover, it was a quite lengthy and elaborate test. To be sure, checking the orientation is not required for every blank separately; it is to be performed on a test piece, parallel to which a whole series of wafers or blanks can be cut out of a crystal block. The result of the former test method would however be that after cutting the test piece the quartz saw would be idle for quite a long time during the lapping and performance of the test.

An economical mass production of oscillator plates with the present extremely strict specifications only became possible upon the introduction of the X-ray diffraction method, with which the crystallographic angles such as XX' of ZZ' can be measured directly, rapidly and very accurately. With the apparatus designed for this purpose by the North American Philips Co., which was used on a large scale in the American quartz industry during the war, the required corrections for a test cut could be determined by relatively unskilled help with an accuracy of a few minutes of arc and the whole measurement required only 10 to 15 seconds time <sup>2)</sup>.

Principle of the angle measurement by X-ray diffraction

The phenomenon of X-ray diffraction in a crystal can be described as a reflection of an X-ray beam at the lattice planes, an appreciable reflection at a set

<sup>2)</sup> W. Parrish and S. G. Gordon, Precise angular control of quartz-cutting by X-rays, Amer. Mineralogist 30, 326-346, 1945. See also W. L. Bond and E. J. Armstrong, Use of X-rays for determining the orientation of quartz crystals, Bell System Tech. J. 22, 293-337, 1943; V. Petržilka and J. Beneš, A method for the determination of crystal cuts by applying the reflection of X-rays from a known lattice plane, Phil. Mag. (7) 37, 399-410, 1946.

of parallel lattice planes only being possible when Bragg's condition,

$$\sin \Theta = \frac{n\lambda}{2d}, \quad \dots \dots \dots (1)$$

is satisfied (see fig. 3). In this relation  $d$  is the spacing of the parallel lattice planes,  $\Theta$  the angle of incidence and emergence between X-ray beam and lattice plane,  $\lambda$  the wavelength of the X-rays,  $n$  a whole number (order of the reflection).

Each set of lattice planes in a quartz crystal makes definite angles with the crystallographic axes ( $X$ ,  $Y$ ,  $Z$ -axes), with the natural faces of the crystal (prism faces  $m$ , major and minor rhombohedron faces  $r$  and  $z$ , respectively, etc.) and therefore also with the various low-temperature coefficient cuts and any auxiliary planes used in the manufacture. Hence, the orientation of a test cut can be checked by measuring the angle between the plane of that cut and a suitably chosen lattice plane. The principle of the arrangement used for this purpose is shown in fig. 4.

The freshly cut flat surface of the piece or wafer cut from the quartz crystal is pressed against the reference surface of the specimen holder which has been ground perfectly plane. The holder has an opening which exposes part of the cut surface. A narrow beam of nearly monochromatic X-rays from the X-ray tube  $B$  passes through collimator slits  $S_1$  and  $S_2$  and impinges on the exposed surface of the wafer. The specimen holder can be rotated about the vertical axis  $P$ . The angle  $\alpha$  between the reference surface and a chosen fiducial or

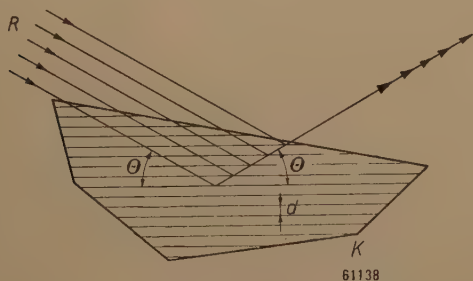


Fig. 3. Diffraction of an X-ray beam  $R$  in a crystal  $K$ . The beam is reflected at a set of parallel lattice planes provided the spacing  $d$  of the planes, the angle  $\Theta$  between the beam and the lattice plane and the wavelength  $\lambda$  of the X-rays satisfy Bragg's condition,  $n\lambda = 2d \sin \Theta$ .

reference position, which for the sake of convenience in this discussion may be chosen in the extension of the primary X-ray beam, can be read off on the goniometer scale. After a lattice plane whose spacing  $d$  and orientation are known has been chosen as a plane of reference, the crystal holder is turned until at the point  $2\Theta$  in the goniometer scale ( $\Theta$  being calculated according to (1) from the known values of  $d$  and  $\lambda$ ) a reflected X-ray beam is observed. The difference between the position  $\alpha$  of the crystal holder and the angle

meter scale ( $\Theta$  being calculated according to (1) from the known values of  $d$  and  $\lambda$ ) a reflected X-ray beam is observed. The difference between the position  $\alpha$  of the crystal holder and the angle

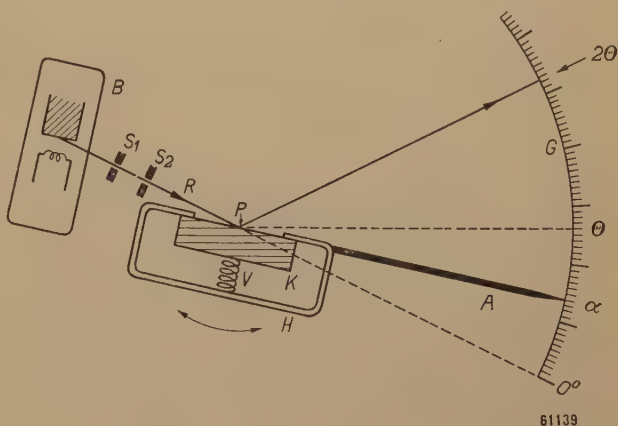


Fig. 4. Principle of the measurement of crystallographic angles by X-ray diffraction.  $H$  specimen holder with reference surface, against which the surface of the crystal piece sawed off as a test cut, or of the test wafer  $K$  is pressed by the spring  $V$ . The holder is fastened to a goniometer arm  $A$  and can be rotated around an axis which at point  $P$  is perpendicular to the plane of the drawing.  $B$  X-ray tube,  $S_1$ ,  $S_2$  collimator slits,  $R$  X-ray beam which falls at  $P$  on the crystal surface to be examined,  $G$  goniometer scale.

$\Theta$  is, as may be seen immediately in fig. 4, the angle between the cutting plane and the lattice plane of reference. (This is rigorously correct only if the lattice plane of reference also is perpendicular to the plane of the drawing; we shall return later to this essential condition.)

### The X-ray diffraction apparatus

In the practical execution of the measurement by this principle the way in which the reflected X-ray beam is observed requires the most attention. In the Philips apparatus a Geiger counter tube is used, in a form which was developed by Friedman<sup>3)</sup> and is shown in fig. 5. The slit in front of the window of the tube is brought to the desired position of the goniometer scale ( $2\Theta$ ) and the tube remains fixed during the measurement. The X-ray quanta entering the slit are detected by current pulses in the tube and the X-ray intensity is indicated directly by use of circuits measuring the average current developed in the tube. We need not go more deeply into the construction or mechanism of the counter tube since these

<sup>3)</sup> H. Friedman, F. K. Kaiser and A. L. Christenson, Applications of Geiger-Muller counters to inspection with X-rays and gamma rays, J. Amer. Soc. Naval Eng. **54**, 177-209, 1942, H. Friedman, Geiger counter tubes, Proc. Inst. Rad. Eng. **37**, 781-808, 1949.

have already been discussed by Friedman<sup>3)</sup> and in this periodical<sup>4)</sup>.

This method of indication has been essential for the success of the apparatus. Photographic detection of the diffraction (which would have required a different X-ray method) would have meant a serious obstruction in the smooth flow of the mass production process because of the time lost in

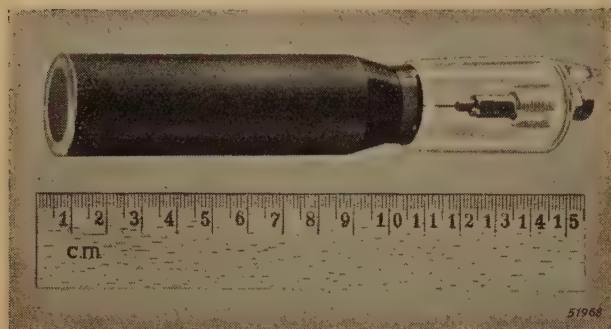


Fig. 5. Geiger counter tube of a recent design. The form originally used in the diffraction apparatus here described was somewhat different but the principle is the same: the X-ray beam to be detected, coming from the left, enters through a thin window of material with low absorption (Lindemann glass, at present mica), and traverses the tube in longitudinal direction running parallel to an anode wire stretched along the axis. The fraction of the radiation absorbed in the argon with which the tube is filled is dependent upon the pressure. Each X-ray quantum absorbed results in a current impulse through the tube between the anode wire and the surrounding cathode cylinder forming the wall of the tube.

developing the film. The counter tube on the other hand records directly the X-radiation falling on the entrance slit. Compared with the fluorescent screen (which was used in only one plant) as means of indication, the counter has the advantages that it is much more sensitive and convenient and that it can be used in a normally lighted room. The ionization chamber which is also used in some apparatus as a direct reading instrument is also less sensitive and less easy to use than the Geiger counter.

In this case the counter tube was used as a so-called proportional counter. The amplification factor of the counter tube in a certain region of voltages (the "plateau") is quite independent of the voltage applied between the wire and cylindrical shell. In a lower voltage region the size of the current impulse caused by one absorbed X-ray quantum is much smaller and increases with the voltage applied and therefore it is possible to

regulate the magnitude of the average current obtained through the counter tube simply by changing the supply voltage. This is a convenience in this application because it makes it possible to keep the response to the wide range of reflection intensities from different lattice planes within the range of the meter. In practice it is not necessary to adjust the voltage when making successive measurements on the same type of cut, using always the same lattice plane of reference.

This method of operation is possible here because a single crystal produces very intense reflections, making it unnecessary to resort to the maximum gain of the counter tube, which is attained when working on the "plateau". (Strictly speaking, the name "Geiger counter" is historically correct only under the latter working condition.) Moreover, in this problem it is only a question of detection of a single diffraction peak. For the apparatus described in<sup>4)</sup>, with which intensities of a number of (much weaker) diffraction lines of powder specimens must be compared, it is important not only to have maximum gain but also to keep the gain of the counter as constant as possible, and hence the "plateau" is used.

Incidentally, it should be noted that the proportional counter has a much shorter "dead time" than has the Geiger counter proper. Therefore, individual quanta may be detected at much higher counting rates, which means that the response of the proportional counter is linear to higher X-ray intensities than that of the Geiger counter.

The slit mounted in front of the window of the counter tube, through which the X-rays pass, is quite wide, so that the setting of the counter tube on the goniometer scale is not critical. This is important because the diffraction spots given by the quartz crystals are very sharp (compared with the diffraction lines of powders) and thus with a slightly incorrect placing of a narrow slit the operator would run the risk of turning the crystal holder past the reflecting position without observing any reflection at all. The slit may not of course be so wide that in certain positions it would allow the passage of two neighboring diffraction spots simultaneously and thus lead to mistakes as to the reflecting lattice plane. But a slit width of  $1/4^\circ$ , measured along the goniometer scale, is quite permissible for the lattice planes normally used in this work, as may be seen from the diffraction spectrum of a quartz sample given in fig. 6.

The apparatus is equipped with an X-ray tube of low power, operating with 3 to 4 mA at a peak voltage of 35 kV, and is air cooled. This is feasible because the intensities of the diffraction spots of single crystal plates are considerably higher than those of the diffraction lines of a polycrystalline specimen. Moreover, the counter tube is much more sensitive than the photographic film usually used in dif-

<sup>4)</sup> J. Bleeksma, G. Kloos and H. J. Di Giovanni, An X-ray spectrometer with Geiger counter for measuring powder diffraction patterns, Philips Tech. Rev. 10, 1-12, 1948 (No. 1). The spectrometer described in this article was developed from the special apparatus designed for quartz oscillator-plate manufacture.

fraction research, so that a relatively low intensity of the primary X-ray beam is sufficient. A copper anode is used giving the characteristic copper radiation, of which the  $K\alpha$  lines are used:  $K\alpha_1 = 1.54050 \text{ \AA}$ ,  $K\alpha_2 = 1.54434 \text{ \AA}$ ;  $K\alpha$  (weighted) =  $1.5418 \text{ \AA}$ . The  $\text{CuK}\beta$ -radiation, which gives diffraction spots at slightly different angles, is sufficiently weakened by a nickel foil in front of the window of the X-ray tube, so that it is not detected.

The opening in the specimen holder must be large enough that the metal edges are not struck

The goniometer scale is graduated in whole degrees, and a fine-adjustment knob which drives the arm of the specimen holder carries a scale graduated in minutes. In order to avoid repeated subtraction of the angle  $\theta$  for a reference lattice plane the goniometer has a second sliding scale whose zero point can be set at the angle  $\theta$  or any other desired fiducial point; this is best done empirically, by means of a standard crystal which has a face lapped parallel to the desired lattice plane. The minute scale on the adjustment knob



Fig. 6. X-ray diffraction spectrum of the front reflection region of a quartz powder sample made with the Geiger counter X-ray spectrometer<sup>5)</sup> (improved form of the instrument previously described in this periodical<sup>4)</sup>). The diffraction spots from single crystal quartz plates are even sharper than the lines of the polycrystalline specimen. This pattern was recorded automatically at a rate of  $\frac{1}{4}^\circ$  of  $2\theta$  per minute, with the copper target X-ray tube operated with full-wave rectification at 40 kV<sub>p</sub>, 20 mA, 0.015 mm nickel filter. The goniometer radius was 170 mm, the angular aperture of the incident beam  $1^\circ$  and width of the receiving slit 0.08 mm. (The 1011-peak is far off the chart.)

by the primary X-ray beam, otherwise X-radiation scattered at the edges would cause a troublesome background intensity in all positions of the counter tube. Furthermore it is important that the reference surface, against which in a smoothly running manufacturing process a block of extremely hard quartz is laid several hundred times daily, should not gradually be worn off and lose its plane surface or its precise setting with respect to the goniometer arm. The holder is therefore made of the hardest available hardened tool steel or of boron carbide, and the reference surface is checked every few days.

can be set at the desired position at  $0'$  by temporarily uncoupling it from the driving gear. The specimen holder can also be unlocked and rotated to any position with respect to the arm which rocks it on the goniometer scale so that one can make measurements on any convenient part of the scale.

A photograph of the arrangement, including the X-ray tube, slit system, shutter, specimen holder, counter tube, direct beam shield, goniometer, meter for reading the X-ray intensity, and supply voltage control for counter tube is shown in fig. 7. The X-ray tube has two windows. Two complete measuring setups can therefore be used, one on either side of the tube. The complete apparatus is shown in fig. 8.

<sup>5)</sup> W. Parrish, X-ray powder diffraction analysis: film and Geiger counter techniques, Science **110**, 368-371, 1949.

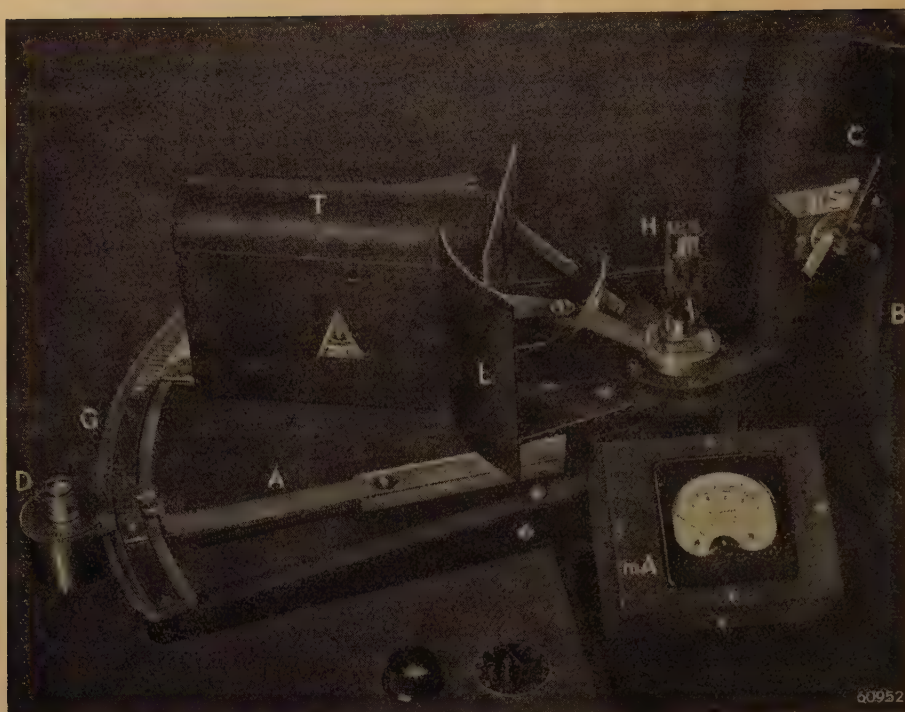


Fig. 7. Measuring table of the X-ray diffraction apparatus. On the right is the X-ray tube housing *B* with slits *S* and a hinged shutter in front of the X-ray tube window. When this shutter is opened a cover *C* descends in front of the crystal holder, so that the operator cannot place his hands in the path of the X-ray beam. On the left is the goniometer scale *G* along which the Geiger counter tube *T* and the goniometer arm *A* bearing the specimen holder *H* can be rocked. The counter tube remains radially directed since it is fastened to an arm which rotates around the same axis (*P* in fig. 4) as the goniometer arm. The crystal holder visible in the middle is intended for measuring blanks; for Y-Z test cuts or for test wafers from an X-block, etc. different holders are used. The undiffracted portion of the primary beam is absorbed by a lead plate *L* behind the crystal holder. On the goniometer scale there is a short sliding auxiliary scale whose position is set with a standard crystal and which permits direct reading of the desired angles. At the end of the goniometer arm is the knob *D* for fine adjustment with scale in minutes. In front is the milliammeter *mA* which indicates the intensity by reading the average current flowing through the counter tube. (The arrangement in this case is the mirror image of that in fig. 4.)

Recently, a number of refinements have been incorporated in the instrument that markedly increase the precision. The width of the diffraction spot and hence the accuracy of the angle measurement, in case of a nearly perfect crystal such as quartz, depends on the width of the source slit  $S_1$  (fig. 4) and the divergence of the X-ray beam falling on the crystal surface. In the instrument described above the slit-width was 0.39 mm and the divergence  $0.9^\circ$ . In the improved arrangement an X-ray tube with a smaller focal spot is used so that it can assume the function of the source slit. This focal spot is 3 mm wide and the beam is obtained at an angle of  $1/4^\circ$  with the anode surface, the projected width thus being 0.04 mm. The divergence is limited to less than  $1'$ .

These changes make it possible to measure angles accurately to within  $0.002^\circ$  to  $0.003^\circ$ , i.e. about 10 seconds. In order to enable the angles to be read with such a precision, the minute scale has been provided with a vernier.

This great accuracy is not necessary for the production of quartz plates. However, it affords the possibility of using the instrument for studying the degree of perfection of crystalline surfaces, say the effect of lapping a crystal plate, and for similar problems.

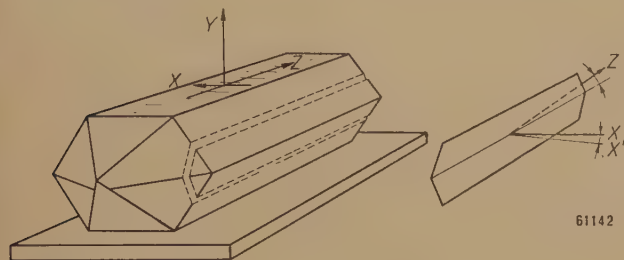
#### Performance of the measurements

In article I it was shown that the desired low-temperature coefficient cut in a quartz block was not obtained in a single operation but in several

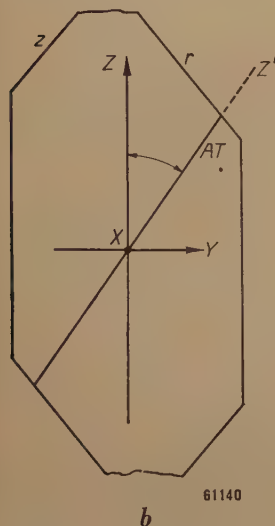


Fig. 8. The complete X-ray diffraction apparatus of the North American Philips Co. for quartz-plate manufacture. The instrument is provided with two independent measuring tables which can be seen here to the left and right of the X-ray tube. Under each measuring table is the supply apparatus for the counter tube, in the middle cabinet the supply and controls for the X-ray tube.

steps. We shall briefly review the way in which the angle measurement with X-ray diffraction enters into the procedure, using the X-block method as an example.



a



b

Fig. 9. a) Cutting a Y-Z-plane from a raw quartz crystal to make an X-block. A prism face (X-Z-plane) of the crystal lies on the saw table, thus an X-axis is horizontal. The Z-axis must be orientated so that it is parallel to the saw blade. An inaccuracy in this orientation leads to a deviation  $XX'$  of the oscillator-plates to be cut out of the block. b) The AT and BT plates are sawn out of the X-block perpendicular to the Y-Z plane which lies in the plane of the drawing. The horizontal Z-axis must be orientated at the prescribed angle to the saw blade. Any inaccuracy in this angle causes deviations of the angle  $ZZ'$  of the oscillator plates.

In the X-block method a plane which should be perpendicular to an X-axis (YZ-plane) is first cut off the quartz crystal. One of the prism faces of the crystal (XZ-plane) is laid on the horizontal saw table (so that an X-axis is horizontal) and a segment is sawn off as nearly as possible parallel to the Z-axis, see fig. 9a. A deviation of parallelism to Z entails a deviation of the normal of the cut surface from the X-axis in the horizontal plane and is found later as angle  $XX'$  in all the blanks obtained from this X-block. It is thus necessary even at this stage to measure the cutting direction accurately and to correct according to the results of the measurement.

The cut YZ-plane of the quartz block is then laid on the saw table (X-axis vertical), which is rotated through an angle read from the degree scale on the table until the saw blade makes the desired angle with the horizontal Z-axis of the quartz block; see fig. 9b. The inaccuracy of this angle setting, when the block is cut into parallel wafers, would occur in all the blanks as an error in the angle  $ZZ'$ . Therefore in this step a test wafer must also be cut, the angle in question measured and the saw should be corrected if necessary.

### Checking a YZ-plane

The YZ-plane which is first cut in the X-block method is itself a lattice plane of the crystal, namely the  $11\bar{2}0$ -plane; see fig. 10. The measurement is in this case relatively simple. The counter tube is set at the Bragg angle on the goniometer scale  $2\Theta = 36^\circ 34'$  for this reference plane. When reflection occurs (more precisely: when maximum reflection occurs), the reflecting  $11\bar{2}0$ -plane will be oriented so as to intersect the goniometer scale at the point  $\Theta = 18^\circ 17'$ , whatever may be the orientation of the actual cut we have made. If the surface was cut exactly parallel to a  $11\bar{2}0$ -plane the goniometer arm which bears the specimen holder will be found at the angle  $\alpha = \Theta$  and it will be independent of any rotation of the cut crystal segment in the holder (with the cut surface always against the vertical reference surface). If the cut plane deviates by a small angle  $\epsilon$  from the  $11\bar{2}0$ -plane, an angle  $\alpha$  is found which is not in general equal to  $\Theta$  (deviation  $\Delta\alpha$ ) and which more-

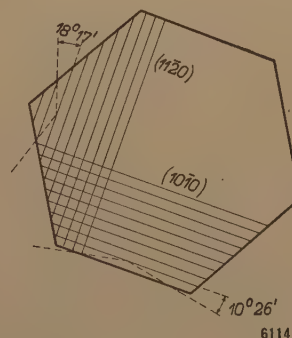
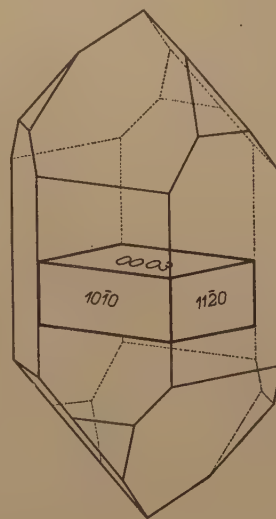


Fig. 10. Position of the  $10\bar{1}0$ ,  $11\bar{2}0$ , and  $0003$ -planes in a quartz crystal; these lattice planes are identical, respectively, with an X-Z plane (prism face), Y-Z plane and X-Y plane. In the cross section dotted lines show the direction of incidence and emergence of X-rays upon reflection for each set of lattice planes.

over varies upon turning the crystal in the holder. In turning the crystal the normal of the reflecting 1120-plane describes a cone of half apex angle  $\varepsilon$  about the horizontally directed normal to the cutting plane (reference surface). For the highest and lowest positions of the normal,  $\Delta a$  equals 0 (unless  $\varepsilon$  is so large that in these positions no reflected intensity at all is observed in the goniometer plane and hence measurement is impossible); for the extreme right- and left-hand horizontal positions of the normal (thus where the 1120-plane of reference is vertical),  $\Delta a = +\varepsilon$  and  $-\varepsilon$  respectively (fig. 11).

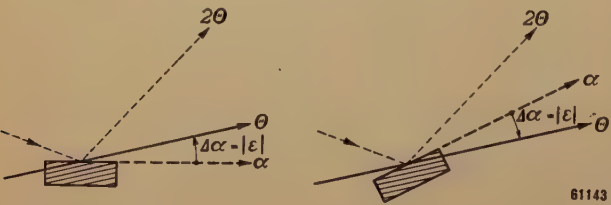


Fig. 11. X-ray reflection is observed when the lattice plane of reference is in the position  $\Theta$ . This corresponds to a position  $\alpha$  of the cut surface of the crystal segment (reference surface of the crystal holder), if the cut surface does not coincide with the lattice plane. The deviation  $\Delta a$ , by which  $\alpha$  differs from  $\Theta$ , varies when the crystal is rotated in the holder; the maximum value of  $\Delta a$ , which is found in two opposite positions of the crystal, equals the correction  $\varepsilon$  of the cutting angle required under the saw to make the cut coincide with the lattice plane.

From the above it will be clear that the angle  $\Delta a$ , which is read off the scale, is equal to the required horizontal correction angle under the saw only when the test cut is placed in the specimen holder in the same orientation it had on the saw table. This is most easily accomplished by drawing a vertical arrow pointed up on the outer surface of the test cut before removing it from the saw table. The test cut is placed in the specimen holder with the fresh cut surface toward the X-ray beam and the arrow again vertical and pointed up. It is then easy also to correlate the direction of the correction: if the goniometer arm must be rotated clockwise from the position  $\Theta$  in order to bring the 1120 reference plane of the test cut into a reflecting position, the crystal on the saw table must also be rotated clockwise in order to make the 1120-plane parallel to the saw block.

It is easy to verify that the correlation thus expressed is valid for the right-hand as well as for the left-hand goniometer in fig. 8, the two scales of which are mirror images of each other. If in cutting a crystal for making an X-block the saw blade was not exactly perpendicular to the saw table, the normal of the cut surface will deviate from the X-axis in the vertical plane. This vertical deviation can be measured in exactly the same way as above, with the only difference that the arrow

painted vertically on the test cut must be placed in a horizontal position. Correlation of the direction of the measured deviation and the required correction is simple also in this case, provided care is taken to note if the test cut was made from the right or left side of the crystal.

Such simple correlation rules, which apply to all types of test cuts, are of great importance for mass production, where the saws and X-ray machines are operated by relatively unskilled persons.

Checking AT and BT cuts

In the previous section the desired cutting plane coincided with a lattice plane, but in checking the low temperature coefficient cuts proper this is not the case. In checking the ZZ' angle of an AT or BT test cut from an X-block therefore a choice must first be made of the lattice plane to be used as plane of reference.

For AT test cuts the 0111-plane (minor rhombohedron face) is chosen as plane of reference. This plane, like the AT cut and most other low-temperature coefficient cuts, is perpendicular to the YZ-plane (see fig. 12), and makes an angle of  $+38^\circ 13'$  with the Z-axis. The ideal AT cut, with  $ZZ' = +35^\circ 15'$ , thus deviates by the angle  $\gamma = -2^\circ 58'$  from the plane of reference. The fact that this deviation

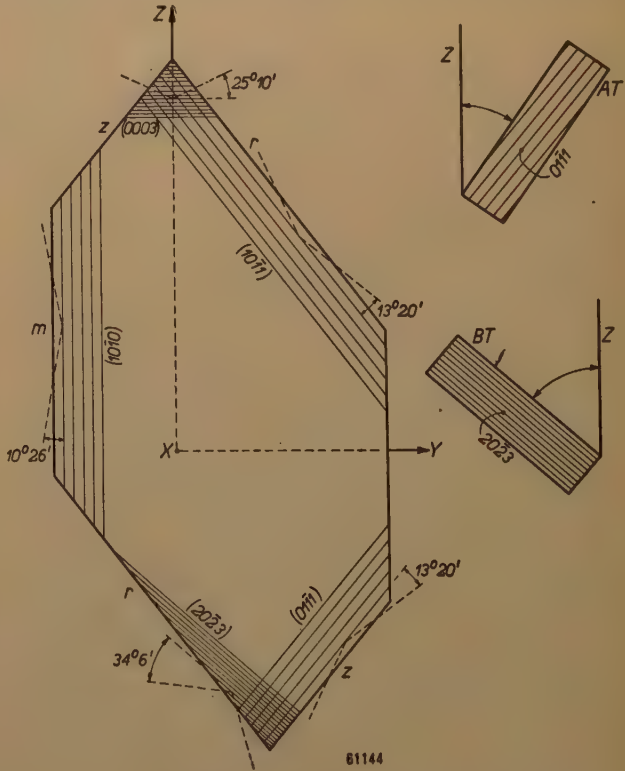


Fig. 12. Position in the quartz crystal of a number of lattice planes all of which are perpendicular to a YZ-plane. The 1011-plane is the major rhombohedron face, the 0111-plane the minor rhombohedron face. The latter is suitable as lattice plane of reference for AT cuts. For BT cuts the 2023-plane is usually used for reference, since it is almost parallel to the BT cut.

Table I. Crystallographic and X-ray data for checking various cuts.

| Name of cut | ZZ' of cut | Reference lattice plane | ZZ' of reference lattice plane | Angle $\gamma$ | Counter tube setting ( $2\theta$ ) for CuK $\alpha$ *) |
|-------------|------------|-------------------------|--------------------------------|----------------|--------------------------------------------------------|
| XZ-plane    | 0°         | 10 $\bar{1}$ 0          | 0°                             | 0°             | 20° 52 $\frac{1}{2}$ '                                 |
| YZ-plane    | 0°         | 11 $\bar{2}$ 0          | 0°                             | 0°             | 36° 34 $\frac{1}{4}$ '                                 |
| AT          | +35°15'    | 01 $\bar{1}$ 1          | +38°13'                        | -2°58'         | 26° 39 $\frac{3}{4}$ '                                 |
| BT          | -49°20'    | 10 $\bar{1}$ 1          | -38°13'                        | -11°07'        | 26° 39 $\frac{3}{4}$ '                                 |
| BT          | -49°20'    | 20 $\bar{2}$ 3          | -49°44'                        | +0°24'         | 68° 12 $\frac{3}{4}$ '                                 |
| CT          | +38°0'     | 01 $\bar{1}$ 1          | +38°13'                        | -0°13'         | 26° 39 $\frac{3}{4}$ '                                 |
| DT          | -51°58'    | 20 $\bar{2}$ 3          | -49°44'                        | -2°14'         | 68° 12 $\frac{3}{4}$ '                                 |
| GT          | +51°08'    | 02 $\bar{2}$ 3          | +49°44'                        | +1°24'         | 68° 12 $\frac{3}{4}$ '                                 |

\*) Based upon CuK $\alpha$  = 1.5418 Å. Calculated from the dimensions  $a$  = 4.9131 Å and  $c$  = 5.4046 Å of the unit cell of the quartz crystal, at 18.0 °C (measurements by A. J. C. Wilson and H. Lipson, Proc. Phys. Soc. 53, 245-250, 1941).

is so small is very important for the practical measurements, as will be seen below. The chief advantages are that even fairly large errors in the cutting plane become accessible to measurement and the accuracy with which the correction can be determined is high.

Imagine the test cut turned in all directions in the crystal holder, as in the previous section, with the cut surface always against the reference surface and with the counter tube fixed in the  $2\theta$  position corresponding to the reflection of the chosen lattice plane of reference. The normal to this reflecting plane again describes a cone but now with a half apex angle  $|\gamma - \epsilon|$ , where  $\epsilon$  is the error in  $ZZ'$  of the test cut. The position  $\alpha$  of the goniometer arm necessary for maximum reflection then varies around the theoretical angle  $\theta$  to an amount between the maximum values  $+|\gamma - \epsilon|$  and  $-|\gamma - \epsilon|$ . In order to measure the correct value of  $|\gamma - \epsilon|$ , an arrow must be used again to preserve the orientation of the test wafer in transferring it from the saw table to the specimen holder of the X-ray machine. On placing the arrow point in the same direction in the holder as under the saw, the simple correlation rule for the direction of the required correction may also be applied <sup>6)</sup>. Since, however, the half apex angle  $|\gamma - \epsilon|$  of the cone may be much larger (nearly equal to  $\gamma$  in the case of small  $\epsilon$ ) than that with which we were concerned in the previous section, small inaccuracies in the position of the arrow when measuring have greater con-

sequences and may cause appreciable errors in the derived correction, errors increasing as  $\gamma$  becomes larger. Moreover, if  $\gamma$  is large, the normal to the lattice plane of reference may deviate so far from the horizontal plane when the arrow is slightly oblique that no reflection is observed in the horizontal goniometer plane and thus no measurement can be made.

From this it follows that it is important to have a small  $\gamma$ , and thus to choose a lattice plane of reference that is as nearly as possible parallel to the cut being checked. For BT cuts, the 20 $\bar{2}$ 3-plane is excellently suited, as this has an angle  $ZZ' = -49^\circ 44'$ ; thus  $\gamma$  is only  $0^\circ 24'$ . Some other lattice planes of reference used for different types of cuts are indicated in table I.

Primarily, the X-ray measurement described above gives only the angle  $|\gamma - \epsilon|$ , indicating the amount and, by the correlation rule, the direction of the correction  $\Delta\alpha$  that would be necessary under the saw to make the test cut coincide with the lattice plane of reference. Now, in order to be able to make the test cut coincide with the ideal AT cut, we must calculate the angle  $\epsilon$ . This is not possible unambiguously from the data obtained: although the value of the angle  $\gamma$  between lattice plane and AT cut is known ( $\gamma = -2^\circ 58'$  in our case), we do not know whether  $\gamma$  should be added to or subtracted from  $\Delta\alpha$ , as the measurement cannot reveal whether  $\epsilon > \gamma$  or  $\epsilon < \gamma$ . This difficulty, which is illustrated by fig. 13 a, b, does not present itself in the case that  $\gamma$  is rather large (lattice plane of reference far from parallelism to AT and to test cut), as in that case there will be no doubt that the orientation error  $\epsilon$  is smaller than  $\gamma$ . If, however, for the sake of precision as explained above,  $\gamma$  is chosen rather small, the ambiguity can be solved only by repeating the measurement of the test cut (in a rough way) using another

<sup>6)</sup> This is the reason for determining on the test wafer the line of the X-axis, which is vertical in the process of cutting, and the direction of the Z-axis, indicating the latter by an arrowhead, as was mentioned in article I. The line of the X-axis is made to stand vertically in the crystal holder, while the Z-arrowhead pointing to the right or to the left gives the correct direction. (The separate blanks are marked in the same way and may be similarly measured to make certain that the blanks to be lapped have an orientation within the desired tolerances.)

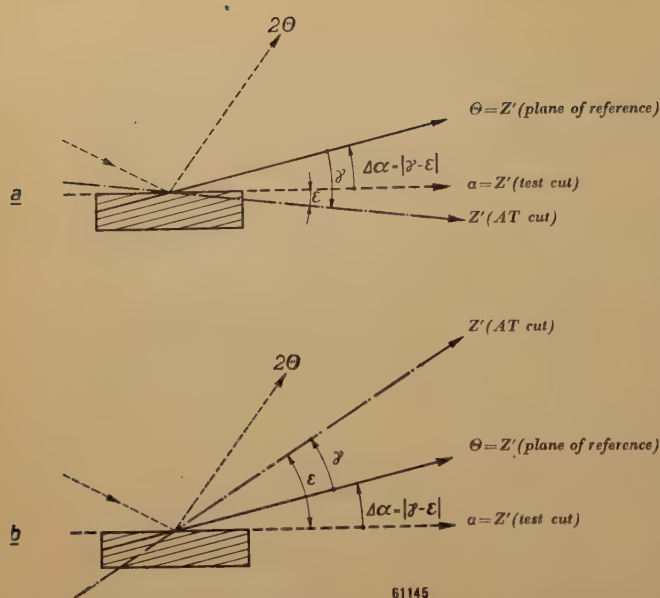


Fig. 13. The X-ray measurement gives the value and direction of the correction angle  $\Delta\alpha = |\gamma - \varepsilon|$  that is required to make a test cut coincide with the lattice plane of reference. In order to find the correction  $\varepsilon$  necessary for obtaining the desired AT cut, it must moreover be known whether  $\gamma > \varepsilon$  or  $\gamma < \varepsilon$ , which is not revealed by the X-ray measurement.

lattice plane of reference, for instance the  $02\bar{2}3$ -plane ( $\gamma' = 35^\circ 15' - 49^\circ 44' = -14^\circ 29'$ ), the two conditions  $\varepsilon = \Delta\alpha \pm \gamma$  and  $\varepsilon = \Delta\alpha' \pm \gamma'$  together leaving only one possible value for  $\varepsilon$ .

There are various other procedures possible with this equipment which are adaptable to the particular method of cutting (the previously discussed "strategy" of the cutting). For example in cases where it is desired to have a surface either parallel to or making some small angle with a chosen lattice plane, and the degree of precision required is beyond the accuracy easily attainable with the saw, the following method has proven useful. The crystal mounted in a special jig which

is adjustable in two mutually perpendicular planes is placed in the X-ray beam and by manipulation of the setting screws of the jig it is tilted until the desired reflection occurs. The crystal is then correctly oriented with respect to the reference edges of the jig. The latter is then transferred to a lapping machine and the crystal surface ground to the orientation set by the X-ray machine. This procedure was used for wafering before accurate sawing methods were introduced. It is apparent that, if large numbers of wafers were cut in this manner, the method would waste a large amount of quartz, because thicker cuts are required than in direct sawing.

The method of measuring crystallographic angles by X-ray diffraction has also assumed some importance in other fields. With some modification it may be applied in the orientation of diamonds for wire drawing, and of sapphire needles used in gramophone pick-ups, in the cutting of barium titanate crystals, etc.<sup>7)</sup>

<sup>7)</sup> Nora Wooster, Orientation of diamonds by X-rays, Ind. Diam. Rev. 3, 1-3, 1943. R. H. Gillette and M. H. Jellinek, An instrument for rapid determination of crystal orientation, Rev. sci. Instr. 20, 480-483, 1949 (No. 7).

**Summary.** After an introductory discussion of the tolerances in the manufacture of quartz oscillator-plates, the X-ray diffraction apparatus is described which was constructed by the North American Philips Co., Inc. during the war to make possible a rapid and accurate check of the crystallographic angles of test cuts. A Geiger counter tube is used in this apparatus to detect the reflected X-ray beam. After placing the counter tube in the position  $2\theta$  ( $\theta = \text{Bragg angle}$ ) on a goniometer scale, the holder in which the test piece, test wafer or blank to be examined is clamped is turned until reflection occurs. The reflecting lattice plane is then oriented according to the angle  $\theta$  on the goniometer scale and from this the angles between this lattice plane and the reference surface of the crystal holder (cutting plane of the test cut) can be derived. The execution of the measurements, in particular the choice of a suitable lattice plane of reference is explained by means of several examples; checking the YZ-cut in making an X-block and checking AT and BT cuts.

## ZINC-OXIDE CRYSTALS



When a small piece of zinc is burnt in a flame a white cloud of zinc-oxide is seen to rise, from which flakes up to a few mm in size are precipitated. A very minute flake collected along the edge of a specimen plate shows under the electron microscope a picture of small crystals with a large number of needle-like spurs loosely intertwined. Photograph taken with the Philips 100 kV electron microscope with a magnification of about 32 000 times.

## THE PRACTICAL APPLICATION OF SAMPLING INSPECTION PLANS AND TABLES

by H. C. HAMAKER, J. J. M. TAUDIN CHABOT and F. G. WILLEMZE.

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*For the practical execution of sampling inspection by unskilled factory personnel the Philips Works in Holland have introduced a sampling table from which a suitable and efficient sampling plan can easily be derived for each particular case.*

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### Recapitulation

The operating characteristic of any sampling plan — single, double, or sequential — can be specified with sufficient accuracy by two parameters. As such the point of control ( $p_0$ ) and the relative slope ( $h_0$ ) possess certain advantages, as explained in a previous article<sup>1)</sup>. If a set of definite values of these parameters have been prescribed, the corresponding single sampling plan, specified by its sample size  $n_0$  (= number of items to be inspected) and acceptance number  $c_0$  (maximum number of rejects permitted for acceptance of the lot), can at once be deduced by means of the relations (see II):

$$c_0 = \frac{\pi}{2} h_0^2 - 0.73, \dots \dots \dots (1)$$

$$n_0 = \frac{c_0 + 0.67}{p_0} \dots \dots \dots (2)$$

Should  $n_0$  turn out to be rather high, this can be remedied by using a double sampling plan with the same characteristic ( $p_0, h_0$ ). How to find such a double plan in a simple manner has also been explained in II (see table II in that article).

It might seem as if the problem of finding the most suitable sampling plan fitting a practical situation has thereby been solved: prescribe values of  $p_0$  and  $h_0$ , and  $n_0$  and  $c_0$  follow automatically from (1) and (2).

To simplify matters still further, we may set up a table from which the values of  $n_0$  and  $c_0$  corresponding to all possible sets of values of  $p_0$  and  $h_0$  can be read at once.

However, in practice the matter is not so simple as suggested here. This will be better understood

if we tear ourselves away from the atmosphere of abstract scientific argument in which this research has so far been conducted and transplant ourselves into the factory where sampling inspection has actually to be carried out.

### Point of control and sharpness of inspection

At one end of the factory material is received, sometimes as raw materials but more often in the shape of parts or components manufactured elsewhere. This material will then be subjected to further treatment or assembled into higher aggregates, until the finished product is delivered at the other end. Frequently also batches of products have to be passed from one department to another during this production process.

For technical and administrative reasons this flow of material is subdivided into definite lots, and our aim will be from time to time to assess the quality of these lots by sampling inspection. This may be done at the entrance, at the exit, between departments, but also during processing, for example on the machine or the conveyor belt.

This last type of inspection, though of great importance because it serves as a check on the production process in the most direct way, is not considered in this paper, where we shall mainly be concerned with incoming and outgoing inspection procedures. Why these two types of inspection should be separately treated is explained later.

The inspection of the incoming and outgoing material will preferably be entrusted to a separate department with specialized personnel. This has the advantage that the useful information supplied by the inspection records with regard to the quality of the products or the manufacturing process can be systematically collected and evaluated.

In consultation with producer and consumer the inspector has to decide what sampling plan is to be applied for each lot of material submitted

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<sup>1)</sup> H. C. Hamaker, The theory of sampling inspection plans, Philips Techn. Rev. 11, 260-270, 1949/1950 (No. 9). See also: H. C. Hamaker, Lot inspection by sampling, Philips Techn. Rev. 11, 176-182, 1949/1950 (No. 6). These two articles will be cited as II and as I respectively.

for inspection. That is to say, he has to choose the point of control and the degree of sharpness (which may suitably be measured by  $h_0$ ) of the inspection procedure.

The point of control may conveniently be interpreted as the point dividing "good" and "bad" lots. Experience has taught that producer and consumer readily agree as to a suitable choice of this parameter.

But the choice of the sharpness of inspection is less simple and straightforward. A sharp inspection requires, roughly, a large sample and consequently much work, and this factor must be balanced against the evil consequences of the erroneous acceptance or rejection of some of the lots as a result of a less sharp control (see I). Even if we restrict ourselves to one type of product, the successive lots coming up for inspection are not of the same size; it commonly happens that one day a lot of 500 pieces has to be checked, and a few days later one of 5000. And it is obvious that in such cases we should wish to exercise a somewhat closer control over the larger lot. If, for example, the point of control has been fixed at 2% (that is to say, a lot with 2% defectives is given a 50% chance of acceptance), then the probability of accepting a lot with 5% defectives will certainly be small; and we shall require this latter probability to be smaller for a lot of 5000 items than for a lot of 500; this can only be achieved by combining the point of control of 2% with a higher value of  $h_0$  in the case of the larger lot.

When the lots submitted for inspection are of the same size, but the points of control vary, a difference in sharpness of inspection is likewise desirable. If the relative slope  $h_0$  were set at the same value with  $p_0 = 2\%$  as with a  $p_0 = 4\%$ , for example, this would require a sample size half as large in the second case. The following arguments show that this cannot as a rule be the best choice. Frequently rejected lots are subjected to a 100% inspection, and sampling inspection necessarily implies that this occasionally happens to some "good" lots. In appraising the advantages of a sampling procedure the additional labour involved should also be taken into account; that is the total work of inspecting both the samples and the rejected "good" lots should be reduced to a minimum.

Now for two different products inspected with  $p_0 = 2\%$  and  $p_0 = 4\%$  respectively but with the same  $h_0$ , the probabilities of unnecessarily having to inspect a good lot will not on the average differ widely and the labour of inspection will be about the same in both cases. The sample size,

however, is half as large in the second case. Consequently if  $h_0$  has been so adjusted that the total amount of inspectional labour (samples + rejected good lots) is a minimum at  $p_0 = 2\%$ , the balance will be upset at  $p_0 = 4\%$  and it will be of advantage to reduce the probability of rejecting a good lot by taking a somewhat higher value of  $h_0$ , that is by using a somewhat larger sample size.

The statement regarding the superfluous sorting out of "good" lots requires some further comment. In passing over from an operating characteristic with  $p_0 = 2\%$  to one with  $p_0 = 4\%$ ,  $h_0$  being unaltered, the probability of rejecting a good lot, with say 1% defectives, is considerably reduced, as will be seen from fig. 1a. But this effect is in practice counter-balanced by the fact that a higher value of  $p_0$  is as a rule assigned to products containing on the average a greater percentage of defectives. It is therefore to be expected that

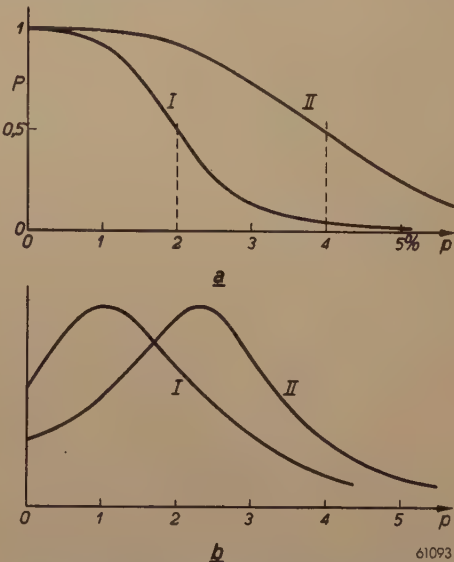


Fig. 1. a) Two operating characteristics with the same relative slope; I has a point of control (= percentage defectives corresponding to a probability of acceptance  $P = \frac{1}{2}$ )  $p_0 = 2\%$ , and II  $p_0 = 4\%$ ,  $h_0$  being equal; the difference amounts to a change of the scale along the abscissa. b) A product for which  $p_0$  is fixed at 4% is in general produced with a higher percentage defective than a product for which  $p_0$  is set at 2%. The relative frequency of occurrence of lots containing  $p\%$  defectives will roughly be as curves I and II in these two cases.

when  $p_0 = 4\%$  then lots with 2% defectives will occur with the same relative frequency as lots with 1% defectives occur when  $p_0 = 2\%$  (cf fig. 1b).

Extended over all percentages less than  $p_0$ , the integral of the product of this relative frequency and the probability of acceptance measures the chance that a "good" lot has to be subjected to 100% inspection. This integrated probability will not differ greatly in the two cases depicted in fig. 1.

So far we have exclusively been arguing from the inspector's point of view, whose main aim is to reduce the labour involved in inspection as much as possible. From the standpoint of the

consumer, however, it is also advisable to increase  $h_0$  as  $p_0$  increases, even for lots of the same size. Leaving  $h_0$  constant, an increase in  $p_0$  signifies a stretching of the operating characteristic by a proportional change in the scale along the  $p$ -axis (fig. 1a), thereby increasing the 10% consumer's risk point in the same ratio as  $p_0$ . Consequently the consumer who was running a 10% risk of receiving a lot with a certain high percentage defectives  $x$  as long as  $p_0$  was 2% will at  $p_0 = 4\%$  run the same 10% risk of obtaining a lot with  $2x$  percent defectives. But the loss that will be incurred owing to the defective products is in this case twice as large. Though the consumer may concede to an increase in  $p_0$  he will not be prepared to accept a proportional increase in his 10% risk point, so that again the use of a steeper operating characteristic, or of a higher value of  $h_0$ , is required.

Summing up, we may conclude that in practice the sharpness of inspection as measured by  $h_0$  must be made a function of both the lot size and the point of control. Such a function, once adopted, might be presented in the form of a table with lot size and point of control as the two main entrances. It is simpler still, however, to combine this procedure with equations (1) and (2); this will lead to another table from which for a given size and point of control the sample size  $n_0$  and acceptance number  $c_0$  can be read off at once.

Only when presented in this form will a sampling table be acceptable in the factory where simplicity of manipulation is a paramount requirement and where even the simplest calculations should preferably be avoided.

### The Philips SSS table

How the relative slope,  $h_0$ , should be made to depend on lot size and point of control is a matter of experience. Precise rules cannot be laid down owing to the impossibility of determining the influencing economic factors, as discussed in I.

Nevertheless there are many cases in modern mass production where these economic factors do not differ greatly in importance where the occasional acceptance of a bad lot does not involve an excessive loss and where the cost of inspection — for example the checking of dimensions with gauges, or the visual inspection of the finish of machine-shop products — is relatively low. As proved by experience, it is possible to provide a single table that can successfully be applied to all such cases alike. The practicability of sampling inspection

is thereby much improved, while at the same time the uniformity attained in taking decisions greatly contributes towards a good understanding and smooth arrangements between consumer and producer.

The Philips Standard Sampling System is a table of this kind which is employed in the Dutch Philips factories and is reproduced in *table I*.

The increase in the relative slope  $h_0$  downwards and towards the right in this table has been chosen in analogy with existing tables which were known to give satisfactory results in practice, particularly in analogy with the Philips R.M.I. Tables (Raw Material Inspection) in use in our factories in Great Britain. Since approximately the same operating characteristics have been adopted as corresponding to that table, both these tables can be interchanged without fear of running into serious trouble.

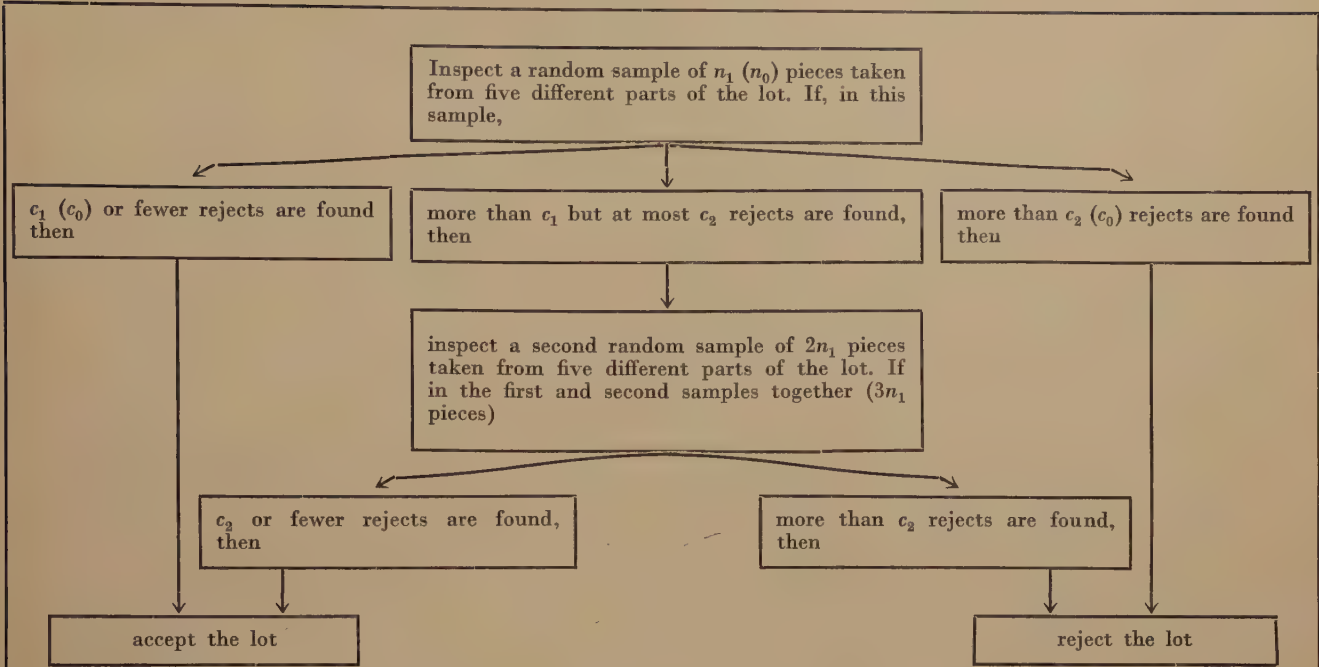
Practically all existing sampling tables are based on the same principle of giving  $n_0$  and  $c_0$  — or in case of double sampling  $n_1, n_2, c_1, c_2$  and  $c_3$  — as a function of two parameters, one of which is always the lot size. Usually the second parameter is not the point of control but some other quantity serving the same purpose of taking into account the requirements of producer and consumer. Our reasons for preferring the point of control have already been explained in II; a further advantage of this parameter is that its meaning can be roughly explained as the boundary between "good" and "bad" lots, a definition that is readily understood in the factory.

The "control" used in the Philips R.M.I. tables is much the same as the "point of control" adopted in the SSS tables, though mathematically not so sharply defined. Against the R.M.I. table, however, some other objections may be raised, as will be explained in the appendix, where various other tables are briefly discussed.

The Philips SSS table contains both single and double sampling plans. Small lots necessarily require small samples, and the advantages of double sampling are then of little importance. Only for lot sizes exceeding 1000 has double sampling been applied.

In accordance with the theoretical arguments advanced in II, the second sample is invariably twice as large as the first, while, to simplify matters still further, we have also adhered to the Dodge and Romig principle ( $c_2 = c_3$ , see Appendix). In most cases  $c_3$  has been made 5 times  $c_1$ , a choice that guarantees a satisfactory efficiency of the double sampling plans.

Tabel I. Philips Standard Sampling System (SSS).



N.B. Where the letter A is given under  $n_0$  this means that no sample is to be taken but that the whole lot has to be inspected.

| POINT OF<br>CONTROL<br><br>LOT SIZE |              | 1/4%  |       | 1/2%  |       | 1%    |       | 2%    |       | 3%    |       | 5%    |       | 7%    |       | 10%   |       |       |       |
|-------------------------------------|--------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
|                                     |              | $n_0$ | $c_0$ | $n_0$ | $c_0$ | $n_0$ | $c_0$ | $n_0$ | $c_0$ | $n_0$ | $c_0$ | $n_0$ | $c_0$ | $n_0$ | $c_0$ | $n_0$ | $c_0$ |       |       |
| Single-sampling<br>plans            |              | $n_0$ | $c_0$ | $n_0$ | $c_0$ | $n_0$ | $c_0$ | $n_0$ | $c_0$ | $n_0$ | $c_0$ | $n_0$ | $c_0$ | $n_0$ | $c_0$ | $n_0$ | $c_0$ |       |       |
|                                     | 20-50        | A     | —     | A     | —     | A     | —     | 30    | 0     | 20    | 0     | 13    | 0     | 10    | 0     | 7     | 0     |       |       |
|                                     | 51-100       | A     | —     | A     | —     | 60    | 0     | 30    | 0     | 20    | 0     | 13    | 0     | 10    | 0     | 7     | 0     |       |       |
|                                     | 101-200      | A     | —     | 100   | 0     | 60    | 0     | 35    | 0     | 55    | 1     | 35    | 1     | 25    | 1     | 17    | 1     |       |       |
|                                     | 201-500      | 175   | 0     | 100   | 0     | 135   | 1     | 75    | 1     | 55    | 1     | 35    | 1     | 40    | 2     | 25    | 2     |       |       |
|                                     | 501-1000     | 225   | 0     | 225   | 1     | 150   | 1     | 85    | 1     | 85    | 2     | 55    | 2     | 55    | 3     | 35    | 3     |       |       |
| Double-sampling<br>plans            |              | $n_1$ | $c_1$ | $c_2$ | $n_1$ | $c_1$ | $c_2$ | $n_1$ | $c_1$ | $c_2$ | $n_1$ | $c_1$ | $c_2$ | $n_1$ | $c_1$ | $c_2$ | $n_1$ | $c_1$ | $c_2$ |
|                                     | 1001-2000    | 330   | 0     | 1     | 150   | 0     | 1     | 110   | 0     | 2     | 55    | 0     | 2     | 45    | 0     | 3     | 25    | 0     | 3     |
|                                     | 2001-5000    | 425   | 0     | 2     | 200   | 0     | 2     | 135   | 0     | 3     | 70    | 0     | 3     | 70    | 1     | 5     | 45    | 1     | 5     |
|                                     | 5001-10000   | 525   | 0     | 3     | 260   | 0     | 3     | 220   | 1     | 5     | 110   | 1     | 5     | 125   | 2     | 10    | 75    | 2     | 10    |
|                                     | 10000-20000  | 875   | 1     | 5     | 440   | 1     | 5     | 380   | 2     | 10    | 190   | 2     | 10    | 180   | 3     | 15    | 110   | 3     | 15    |
|                                     | 20000-50000  | 1500  | 2     | 10    | 750   | 2     | 10    | 540   | 3     | 15    | 270   | 3     | 15    | 240   | 4     | 20    | 140   | 4     | 20    |
|                                     | 50000-100000 | 2200  | 3     | 15    | 1100  | 3     | 15    | 700   | 4     | 20    | 390   | 4     | 20    | 290   | 5     | 25    | 175   | 5     | 25    |

Slight discontinuities are unavoidable because the acceptance numbers ( $c_0$  or  $c_1$ ,  $c_2$ ,  $c_3$ ) have always to be rounded off to integer values, but this is of no practical consequence since in the choice of a sampling plan a high precision is not required. For the same reason it is permissible to subdivide the lot sizes and points of control into a few classes only, thus limiting the size of the table so that it

can easily be issued in a pocket-size edition. In classifying lot sizes the well-known sequence 1, 2, 5, 10, 20, 50 etc. has been used, a feature that may help to make the table more readily acceptable to the inspector <sup>2)</sup>. In order to prevent inhomogenieties in the lot causing a bias in the sample it has been prescribed

<sup>2)</sup> See next page.

|                                                                                 |  |                                              |  |                              |  |
|---------------------------------------------------------------------------------|--|----------------------------------------------|--|------------------------------|--|
| INSPECTION SPECIFICATION CARD                                                   |  | ARTICLE: <i>Casing</i>                       |  | CODE NUMBER: <i>20454.91</i> |  |
| SAMPLING TABLE: <i>SSS</i>                                                      |  | CONSUMER: <i>Dept. H</i>                     |  |                              |  |
| POINT OF CONTROL: <i>3%</i>                                                     |  | SUPPLIER: <i>Hendriks</i>                    |  |                              |  |
| COMPILED BY: <i>van Doesburg</i>                                                |  |                                              |  |                              |  |
| Dimensions and properties to be tested                                          |  | Measuring tools                              |  | Remarks                      |  |
| <i>Material: See drawing</i>                                                    |  | <i>Magnet</i>                                |  |                              |  |
| <i>Finishing: No strain grooves, no burring, surface free of spots and dirt</i> |  |                                              |  |                              |  |
| <i>Dimensions: Gauge must pass through right to the bottom</i>                  |  | <i>Gauge 23, slightly greased</i>            |  |                              |  |
| <i>No deviations in size 10 - 0.2</i>                                           |  | <i>Sliding gauge</i>                         |  |                              |  |
| <i>Size 2.5 - 0.1 may not be smaller</i>                                        |  | <i>"</i>                                     |  |                              |  |
| <i>Incision 1.5 must not be shallower</i>                                       |  | <i>Gauge 22</i>                              |  |                              |  |
| <i>Size 8 F<sup>0</sup> true and smooth</i>                                     |  | <i>Pin 8 F<sup>0</sup>, slightly greased</i> |  |                              |  |

Fig. 2. "Inspection specification card" on which the requirements to be satisfied by a particular product have been registered.

that each sample should be composed of at least 5 sub-samples drawn from different places in the lot.

Finally it may be mentioned that the table is only intended as a general directive and not as a rigid rule. If experience shows that the lots of a certain product are practically always accepted and never rejected, the inspector is permitted to reduce his inspection procedure. In such cases he is advised to leave the point of control unaltered and to change over to a sampling plan in the same column of the table one or two lines higher up than the plan so far applied.

2) When the lot is of small size the sample will often embrace a large proportion of the whole lot and the conditions under which formulae (1) and (2) are valid (Poisson probabilities) do not apply. By a more detailed analysis it has been found, however, that equations (1) and (2) can be extended so as to provide a reasonable approximation also under these circumstances. We then have

$$n_0 p_0 = c_0 + 0.67 - 0.33 \frac{n_0}{N},$$
$$(\pi/2) h_0^2 = (n_0 p_0 + 0.06)/(1 - n_0/N),$$

where  $N$  denotes the lot size. These equations still presuppose that  $p_0$  is small. We see that  $h_0$  increases and the operating characteristic thus becomes steeper as the ratio  $n_0/N$  becomes larger;  $h_0$  becomes infinite when  $n_0 = N$ , that is when the whole lot is inspected.

In setting up the SSS table these extended formulae have been used as far as possible. To take the effect of a finite sample size accurately into account it would, however, be necessary to subdivide the lot size into a larger number of classes, and this did not seem advisable.

The use of the table in practice

In addition to one or two copies of the table an inspection department should have at its disposal the complete specification of the requirements to be satisfied by the various articles it may be called upon to inspect.

It is convenient to have these arranged in a card index according to code number, and fig. 2 shows a card as in use in some of the inspection departments of our Eindhoven factories. On this card there are also recorded the tools to be used and the point of control required.

It is moreover of importance carefully to register the results of each inspection carried out, and in fig. 3 a second card designed for this purpose is reproduced. Next to the date and the number of the consignment follow the size of the lot and the result of the inspection by sample, while farther to the right some columns have been provided for recording the decision taken.

Most types of products may show a series of different defects and the actual kinds of defects observed can be specified on the back (fig. 3b). In cases of controversy it is then always possible to produce the data that resulted from the actual inspection.

When materials received from outside have to

| INSPECTION RECORD |                 |                |               |                |               |             |                |               |                            | ARTICLE: <i>Casing</i>      |                | CODE NUMBER: <i>20454-91</i> |                |                 |                         |              |               |
|-------------------|-----------------|----------------|---------------|----------------|---------------|-------------|----------------|---------------|----------------------------|-----------------------------|----------------|------------------------------|----------------|-----------------|-------------------------|--------------|---------------|
|                   |                 |                |               |                |               |             |                |               |                            | SAMPLING TABLE: <i>555</i>  |                |                              |                |                 |                         |              |               |
|                   |                 |                |               |                |               |             |                |               |                            | POINT OF CONTROL: <i>3%</i> |                | SUPPLIER: <i>Hendricks</i>   |                |                 |                         |              |               |
| LOT No.<br>#      | DATE            | ORDER No.<br>3 | LOT SIZE<br>4 | 1st SAMPLE     |               |             | 2nd SAMPLE     |               | Inspector's initials<br>10 | DECISION                    |                |                              |                | SORTING         |                         |              | LOT No.<br>18 |
|                   |                 |                |               | Inspected<br>5 | Rejected<br>6 | 0/10<br>7   | Inspected<br>8 | Rejected<br>9 |                            | Good<br>11                  | Rejected<br>12 | Sorted<br>13                 | Accepted<br>14 | Inspected<br>15 | Rejected<br>Total<br>16 | Rework<br>17 |               |
| 1                 | <i>9/14/40</i>  | <i>4100</i>    | <i>600</i>    | <i>85</i>      | <i>0</i>      |             |                |               | <i>HB</i>                  | <i>600</i>                  |                |                              |                |                 |                         |              | <i>1</i>      |
| 2                 | <i>7/12</i>     | <i>4141</i>    | <i>1073</i>   | <i>45</i>      | <i>1</i>      |             | <i>90</i>      | <i>1</i>      | <i>HB</i>                  | <i>1073</i>                 |                |                              |                |                 |                         |              | <i>2</i>      |
| 3                 | <i>3/12</i>     | <i>5177</i>    | <i>893</i>    | <i>85</i>      | <i>1</i>      |             |                |               | <i>HB</i>                  | <i>893</i>                  |                |                              |                |                 |                         |              | <i>3</i>      |
| 4                 | <i>9/12</i>     | <i>5227</i>    | <i>3260</i>   | <i>70</i>      | <i>2</i>      |             | <i>140</i>     | <i>2</i>      | <i>HB</i>                  | <i>3260</i>                 |                |                              |                |                 |                         |              | <i>4</i>      |
| 5                 | <i>15/12</i>    | <i>6531</i>    | <i>5183</i>   | <i>125</i>     | <i>24</i>     |             |                |               | <i>HB</i>                  |                             | <i>5183</i>    |                              |                |                 |                         |              | <i>5</i>      |
| 6                 | <i>12/12</i>    | <i>6876</i>    | <i>793</i>    | <i>85</i>      | <i>4</i>      |             |                |               | <i>JD</i>                  |                             |                | <i>793</i>                   |                | <i>793</i>      | <i>59</i>               |              | <i>6</i>      |
| 7                 | <i>12/14/40</i> | <i>8331</i>    | <i>2021</i>   | <i>70</i>      | <i>1</i>      |             |                |               | <i>JD</i>                  | <i>2021</i>                 |                |                              |                |                 |                         |              | <i>7</i>      |
| 8                 | <i>12/1</i>     | <i>8744</i>    | <i>2436</i>   | <i>70</i>      | <i>0</i>      |             |                |               | <i>BF</i>                  | <i>2436</i>                 |                |                              |                |                 |                         |              | <i>8</i>      |
| 9                 | <i>18/1</i>     | <i>8721</i>    | <i>1331</i>   | <i>45</i>      | <i>0</i>      |             |                |               | <i>AF</i>                  | <i>1331</i>                 |                |                              |                |                 |                         |              | <i>9</i>      |
| 10                | <i>24/3</i>     | <i>10353</i>   | <i>6028</i>   | <i>125</i>     | <i>5</i>      |             | <i>250</i>     | <i>12</i>     | <i>JD</i>                  |                             |                |                              | <i>6028</i>    |                 |                         |              | <i>10</i>     |
| 11                | <i>25/3</i>     | <i>10358</i>   | <i>979</i>    | <i>85</i>      | <i>1</i>      |             |                |               | <i>HB</i>                  | <i>979</i>                  |                |                              |                |                 |                         |              | <i>11</i>     |
| 12                | <i>28/3</i>     | <i>11461</i>   | <i>1725</i>   | <i>45</i>      | <i>0</i>      |             |                |               | <i>AF</i>                  | <i>1725</i>                 |                |                              |                |                 |                         |              | <i>12</i>     |
| 13                | <i>3/13</i>     | <i>11783</i>   | <i>2666</i>   | <i>70</i>      | <i>13</i>     |             |                |               | <i>JD</i>                  |                             |                | <i>2666</i>                  |                | <i>2666</i>     | <i>497</i>              |              | <i>13</i>     |
| 14                | <i>7/14</i>     | <i>12522</i>   | <i>650</i>    | <i>85</i>      | <i>2</i>      |             |                |               | <i>JD</i>                  | <i>650</i>                  |                |                              |                |                 |                         |              | <i>14</i>     |
| 15                | <i>13/14</i>    | <i>14612</i>   | <i>3579</i>   | <i>70</i>      | <i>0</i>      |             |                |               | <i>HB</i>                  | <i>3579</i>                 |                |                              |                |                 |                         |              | <i>15</i>     |
| 16                |                 | <i>15117</i>   | <i>3711</i>   | <i>70</i>      | <i>0</i>      |             |                |               | <i>HB</i>                  | <i>3711</i>                 |                |                              |                |                 |                         |              | <i>16</i>     |
| 17                | <i>12/14</i>    | <i>16277</i>   | <i>1539</i>   | <i>45</i>      | <i>2</i>      |             | <i>90</i>      | <i>0</i>      | <i>AF</i>                  | <i>1539</i>                 |                |                              |                |                 |                         |              | <i>17</i>     |
| 18                | <i>14/5</i>     | <i>18356</i>   | <i>1967</i>   | <i>45</i>      | <i>1</i>      |             |                |               | <i>AF</i>                  | <i>1967</i>                 |                |                              |                |                 |                         |              | <i>18</i>     |
| 19                | <i>19/5</i>     | <i>19783</i>   | <i>1004</i>   | <i>45</i>      | <i>0</i>      |             |                |               | <i>JD</i>                  | <i>1004</i>                 |                |                              |                |                 |                         |              | <i>19</i>     |
| 20                |                 |                |               |                |               |             |                |               |                            |                             |                |                              |                |                 |                         |              | <i>20</i>     |
| TOTALS            |                 |                | <i>43673</i>  | <i>1435</i>    | <i>59</i>     | <i>4.1%</i> |                |               |                            |                             |                |                              |                |                 |                         |              |               |

| SPECIFICATION OF DEFECTS OBSERVED IN 1st SAMPLE |                                                                                                                                                                                                                                  |   |   |   |    |   |   |   |   |    |    |    |    |    |    |         |
|-------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---|---|---|----|---|---|---|---|----|----|----|----|----|----|---------|
| LOT No                                          | <div>strain<br/>grooves<br/>buried<br/>spotted<br/>dirty<br/>gauge 23<br/>not fitting<br/>size 10-02<br/>too large<br/>size 10-02<br/>too small<br/>size 2.5-0.1<br/>too small<br/>not deep enough<br/>size 0F 8<br/>wrong</div> |   |   |   |    |   |   |   |   |    |    |    |    |    |    | REMARKS |
|                                                 | 1                                                                                                                                                                                                                                | 2 | 3 | 4 | 5  | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |         |
| 1                                               |                                                                                                                                                                                                                                  |   |   |   |    |   |   |   |   |    |    |    |    |    |    |         |
| 2                                               |                                                                                                                                                                                                                                  |   |   |   |    | 1 |   |   |   |    |    |    |    |    |    |         |
| 3                                               |                                                                                                                                                                                                                                  |   |   |   |    |   |   | 1 |   |    |    |    |    |    |    |         |
| 4                                               |                                                                                                                                                                                                                                  |   | 2 |   |    |   |   |   |   |    |    |    |    |    |    |         |
| 5                                               | 24                                                                                                                                                                                                                               |   |   |   |    |   |   |   |   |    |    |    |    |    |    |         |
| 6                                               |                                                                                                                                                                                                                                  |   |   |   | 1  |   |   | 3 |   |    |    |    |    |    |    |         |
| 7                                               |                                                                                                                                                                                                                                  |   |   |   | 1  |   |   |   |   |    |    |    |    |    |    |         |
| 8                                               |                                                                                                                                                                                                                                  |   |   |   |    |   |   |   |   |    |    |    |    |    |    |         |
| 9                                               |                                                                                                                                                                                                                                  |   |   |   |    |   |   |   |   |    |    |    |    |    |    |         |
| 10                                              | 5                                                                                                                                                                                                                                |   |   |   |    |   |   |   |   |    |    |    |    |    |    |         |
| 11                                              |                                                                                                                                                                                                                                  | 1 |   |   |    |   |   |   |   |    |    |    |    |    |    |         |
| 12                                              |                                                                                                                                                                                                                                  |   |   |   |    |   |   |   |   |    |    |    |    |    |    |         |
| 13                                              |                                                                                                                                                                                                                                  |   |   |   | 13 |   |   |   |   |    |    |    |    |    |    |         |
| 14                                              | 2                                                                                                                                                                                                                                |   |   |   |    |   |   |   |   |    |    |    |    |    |    |         |
| 15                                              |                                                                                                                                                                                                                                  |   |   |   |    |   |   |   |   |    |    |    |    |    |    |         |
| 16                                              |                                                                                                                                                                                                                                  |   |   |   |    |   |   |   |   |    |    |    |    |    |    |         |
| 17                                              |                                                                                                                                                                                                                                  | 2 |   |   |    |   |   |   |   |    |    |    |    |    |    |         |
| 18                                              |                                                                                                                                                                                                                                  |   |   |   |    |   |   |   |   |    |    |    |    |    |    |         |
| 19                                              |                                                                                                                                                                                                                                  |   |   |   |    |   |   |   |   |    |    |    |    |    |    |         |
| 20                                              |                                                                                                                                                                                                                                  |   |   |   | 2  |   | 1 |   |   |    |    |    |    |    |    |         |
| TOTAL                                           |                                                                                                                                                                                                                                  |   |   |   |    |   |   |   |   |    |    |    |    |    |    |         |

37 200 68

Fig. 3. "Inspection record" on which, for a particular product, the data resulting from the samples are registered. Top: the front with the results of the successive inspections. Below: the back where the defects observed are analyzed in detail.

be inspected it is of some importance to set up a separate record for each product and for each of the sources of supply if there are more than one. When the record (fig. 3a) has been completed the sample data (in the case of double sampling only those of the first sample) can be added to give a final result. In fig. 3a, for example, 59 rejects have been detected in a total of 1435 items inspected, that is 4.1 %. This provides a measure of the average percentage defectives produced or received during the period under consideration. The final data can be entered on a separate list, and after a certain lapse of time the original records can be destroyed in order to avoid a harassing accumulation of paper.

If the amount of material needed diminishes or increases it may be of advantage to take into account the quality as well as the price of the products from different suppliers, and the collective records will furnish the data required. They will likewise provide useful information for the producer as to what types of defects are of most frequent occurrence and should be particularly attended to.

In conclusion it may be worth while mentioning the limits and restrictions to the use of the SSS table.

One important category to which the table does not apply are life tests, and destructive tests in general. It will be understood that owing to the high cost of inspection the interplay of various economic factors lead in this case to different requirements regarding point of control and relative slope than those on which the SSS table has been founded.

The control of the production process is another case outside the scope of our table. Here the flow of material passing a machine or a conveyor belt is subdivided into small batches consisting, for instance, of half an hour's output. These are tested by small samples, so that serious derangements in the production process, such as may result from sudden misalignment of machines, deterioration of tools or lack of attention of personnel, can be quickly detected and corrected. The discovery of these serious errors does not require a very sharp inspection, so that the sample sizes may be much smaller than in the SSS table. If subsequently the output of say a day or a week is pooled to form one lot, the percentage defectives in this total lot can be judged by adding together the results of all the samples inspected during the corresponding period; in this way a final and sharp control of the lot as a whole can be achieved without additional inspection.

#### Appendix: Other sampling tables

Dodge and Romig were the first to publish, in 1941, a set of four different sampling inspection tables, two for single and two for double sampling. In these tables, or rather series of tables, the choice of a sampling plan is determined by three parameters. It is assumed that by previous observation the average percentage defectives in the lots submitted for inspection has been determined. This so-called "process average" serves to discount the quality that can be produced; it is one of the two parameters functioning as main entrances in each table, the lot size being the other. As third parameter the average outgoing quality limit (AOQL) is employed in two of the series of tables (one for single and one for double sampling), and the 10% consumer's risk point (see II, p. 267/268), called "the lot tolerance percent defective", in the other two. In the double-sampling plans the ratio of the sample sizes varies, but the criteria always satisfy the condition  $c_2 = c_3$ , which we have consequently designated as the Dodge and Romig principle.

Lot size, process average, and 10% consumer's risk point (or AOQL) have been classified into 20, 6 and 8 different classes respectively. Consequently each series of tables comprises  $20 \times 6 \times 8 = 960$  different cases for each of which a sampling plan is prescribed; hence these tables cover many pages.

Since these tables were developed it has been realized that the differentiation had been carried one or two stages further than needed in practice. This at least explains that the "Army Service Forces Tables" — set up during the second world war by a team of statisticians including Dodge and Romig, and later taken over by the Statistical Research Group, Columbia University — are simpler and much more concise than the original tables of Dodge and Romig. Here only two parameters are employed, namely the 5% producer's risk point and the size of the lot. To simplify matters still further the sample size has been once and for all coupled with the lot size; when arranged as in table I the Army Service Forces Tables contain one sample size for each line of the table which has to be used in all columns. Separate tables have been provided for single and for double sampling, while in the latter the second sample is invariably twice as large as the first. A subdivision according to 7 different lot sizes and 14 values of the producer's risk point leads to a table of in all 98 combinations which may be printed on a single page <sup>3)</sup>.

Despite the simplifications, certain objections can still be brought forward against these tables, which, as far as we can see, are mainly a consequence of the rigid connection between lot size and sample size.

As may be inferred from equations (1) and (2), this coupling signifies that for a given lot size (and consequently a fixed value of  $n_0$ ) the acceptance number  $c_0$  and, with it,  $h_0$  are completely prescribed in their dependence of  $p_0$ . For two different lot sizes this functional relation between  $h_0$  and  $p_0$  is represented by the curves CRG in fig. 4.

That  $h_0$  increases with  $p_0$  is in keeping with the principles explained earlier in this paper. In our opinion, however, the

<sup>3)</sup> 500 is the smallest value of lot sizes in the "Army Service Forces Tables". They have been extended by the Statistical Research Group, Columbia University, towards the side of smaller lots, which has raised the number of lot-size classes to 14.

increase of  $h_0$  with  $p_0$  is in the "Army Service Forces Tables" more pronounced than is justifiable on these grounds.  
In the R. M. I. tables of our English factories, and conse-

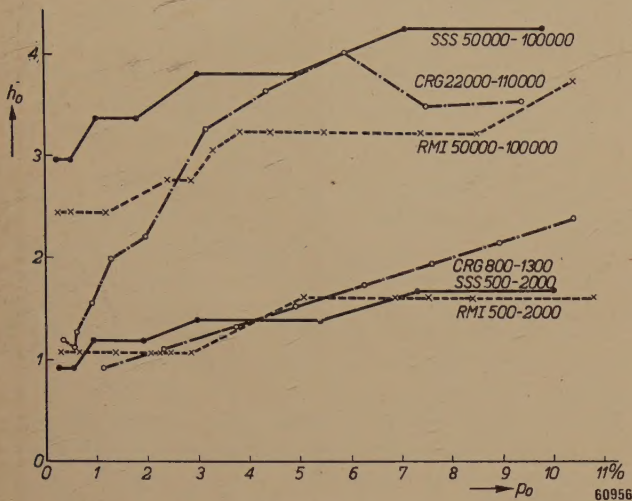


Fig. 4. Variation of  $h_0$  in various sampling tables, plotted as a function of  $p_0$  for a class with a medium and for a class with a large lot size.  
CRG: tables of the Columbia Research Group.  
RMI: Raw Material Inspection table of the English Philips' factories.  
SSS: Philips Standard Sampling System.

The irregularities in the curves are due to the fact that only whole numbers could be chosen for the criteria. The sharp bend in the CRG curve for large lots arises from the fact that in the CRG table at high values of  $p_0$  the coupling of lot size and sample size referred to in the text is no longer adhered to and smaller samples are used.

quently also in our SSS tables, a more gradual increase of  $h_0$  with  $p_0$  has been adopted, which is expressed in table I as a decrease of the sample sizes with increasing  $p_0$  for a fixed lot size. The relation between  $h_0$  and  $p_0$  as adopted in the R. M. I. and SSS tables has also been represented in fig. 4, each by two curves holding for lot sizes approaching those of the CRG curves as closely as possible.

More serious objections may be raised against the double sampling plans adopted in the "Army Service Forces Tables". As inferred in II from the random walk diagram, a reasonable efficiency requires that the criteria of a double-sampling plan shall satisfy the relation.

$$c_1 < \frac{n_1}{n_1 + n_2} c_3 \dots \dots \dots (3)$$

If this condition is not fulfilled we obtain double-sampling plans which operate with an efficiency even worse than that of an equivalent single-sampling plan, as was demonstrated in fig. 6 of II. Now it has been found that out of the 40 different combinations of criteria found in the Army Service Forces Tables no less than 27 do not satisfy equation (3), which in this particular case requires  $c_1 < \frac{1}{3} c_3$ . In constructing this double-sampling table the purpose has been to obtain double-sampling plans with operating characteristics closely approximating those of the corresponding single-sampling plans in the single-sampling table. Since, however, the sample sizes for double sampling were prescribed and coupled with the lot size, the requisite variation in the operating characteristic could only be achieved by having recourse to inefficient combinations of the criteria.

Apart from this, in our opinion the sample sizes have not been correctly fixed. For example, corresponding to a single-sampling plan with

$$n_0 = 150, \quad c_0 = 6$$

the Columbia Research Group arrives at a double sampling plan with

$$n_1 = 100, \quad n_2 = 200, \quad c_1 = 3, \quad c_2 = c_3 = 9,$$

whereas by means of table II in (II) we find

$$n_1 = 61, \quad n_2 = 122, \quad c_1 = 1, \quad c_2 = c_3 = 7$$

as the most suitable approximation;  $h_0$  is 2.07 for the single sampling and 2.08 for this second double-sampling plan, so that the operating characteristics lie very close together. The ratio  $c_1/c_3$  is  $\frac{1}{3}$  in the first and  $\frac{1}{7}$  in the last of the double-sampling plans, the higher efficiency of which is demonstrated by a reduction of about 40 % in the sample size.

Consequently, corresponding to a single sample of 150, the double-sample sizes  $n_1 = 100, n_2 = 200$  are too large, and if we try to adjust the two plans to one operating characteristic we are necessarily forced to adopt an inefficient combination of criteria in the latter case.

It will be appropriate to conclude this paper with a brief discussion of the Philips R.M.I. tables introduced in our English factories by A. S. Wharton. Part of this table is reproduced in Table II.

The R.M.I. system prescribes that not one but five different samples shall be taken all of the same size,  $N$ , and inspected separately. If the number of defectives recorded in each of these five samples is less than the criterion  $K$  the lot is accepted straightaway; if in one sample the number of defectives is greater than  $K$ , or if in two samples it is equal to  $K$ , the lot is rejected. But if in one out of the five samples  $K$  defectives are found, the other samples containing less than  $K$ , then a

Table II. Part of the R.M.I. table as used in the Philips factories in England.

| Point of control<br>Lot size | 1/4% |   | 1/2% |   | 1%  |   | 2%  |   | 2 1/2% |   | 3%  |   |
|------------------------------|------|---|------|---|-----|---|-----|---|--------|---|-----|---|
|                              | N    | K | N    | K | N   | K | N   | K | N      | K | N   | K |
| 500 - 2000                   | 65   | 1 | 30   | 1 | 15  | 1 | 10  | 1 | 9      | 1 | 8   | 1 |
| 2000 - 5000                  | 65   | 1 | 30   | 1 | 15  | 1 | 30  | 2 | 28     | 2 | 25  | 2 |
| 5000 - 15000                 | 290  | 2 | 140  | 2 | 50  | 2 | 30  | 2 | 28     | 2 | 45  | 3 |
| 15000 - 25000                | 540  | 3 | 275  | 3 | 120 | 3 | 60  | 3 | 50     | 3 | 45  | 3 |
| 25000 - 50000                | 540  | 3 | 275  | 3 | 120 | 3 | 90  | 4 | 70     | 4 | 60  | 4 |
| 50000 - 100000               | 800  | 4 | 420  | 4 | 180 | 4 | 120 | 5 | 100    | 5 | 110 | 6 |
| 100000 - 250000              | 1050 | 5 | 560  | 5 | 280 | 5 | 180 | 6 | 140    | 6 | 145 | 7 |

second set of five samples of the same size  $N$  must be drawn, and the lot is finally accepted if all these contain less than  $K$  defectives.

Thus in principle we have again to do with a double-sampling table, judgement being based on the maximum number of defectives observed in a set of five equal samples. This is the weak point; the numbers of defectives found in the other four samples furnish a certain amount of information with respect to the percentage defectives in the lot, and this infor-

mation is discarded. It is therefore to be expected that the sampling plans of the R.M.I. tables will be relatively inefficient<sup>4)</sup>; this is corroborated by the inverse efficiency characteristics in fig. 5, which have been obtained by the methods developed in II.

For  $K > 1$  the efficiency is reasonably good owing to the fact that under these conditions the R.M.I. system operates as a double-sampling plan: whether we require that the number of defectives in every one of five samples shall be less than 1, or that this shall hold for the total number observed in the five samples together, is just the same thing.

For  $K > 1$  the efficiency of the R.M.I. table is not as satisfactory as it might be, which shows that by resorting to single or double-sampling plans it should be possible to diminish the average sample size without impairing the degree of control exercised. This possibility of reducing the average number of observations required, while retaining the practical features of the R.M.I. table, has led us to develop the SSS table described above; as already stated and illustrated in fig. 4, the relation between  $p_0$  and  $h_0$  is approximately the same in both tables and the operating characteristics of corresponding sampling plans consequently lie close together.

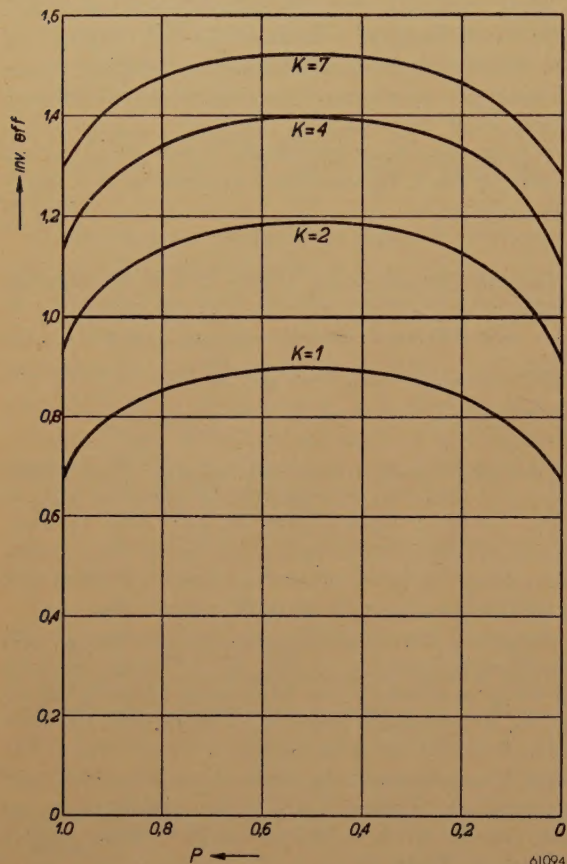


Fig. 5. Inverse efficiency characteristics of the sampling plans of the R.M.I. table. For  $K > 1$  efficiency is less than that of the equivalent single-sampling plans (which, by way of definition, has been given the efficiency 1).

<sup>4)</sup> To avoid misunderstanding it should be borne in mind that the "efficiency" is a mathematical concept referring only to the number of observations needed and not to the other factors, such as conciseness and simplicity, which also contribute towards the practical success of a table in the factory and which have been duly considered above.

**Summary.** If we specify the operating characteristic of a sampling plan by its point of control,  $p_0$ , and relative slope,  $h_0$ , a suitable single or double sampling plan corresponding to any prescribed course of the characteristic can be found in a simple manner. In general it will be advantageous to use a steeper characteristic (a higher value of the relative slope) the larger the size of the inspection lot and the higher the point of control. If a certain functional relation between  $h_0$ ,  $p_0$  and lot size is adopted then a table can be compiled from which a sampling plan can at once be derived as a function of lot size and point of control.

The Philips SSS table, which was mainly developed on these principles, has in other respects been adjusted to practical requirements with a view to its applicability in the factory by unskilled personnel. In an appendix various other sampling tables are compared with the SSS table.

## ABSTRACTS OF RECENT SCIENTIFIC PUBLICATIONS OF THE N.V. PHILIPS' GLOEI-LAMPENFABRIEKEN

Reprints of these papers not marked with an asterisk can be obtained free of charge upon application to the address printed on the back cover.

**1884:** J. L. H. Jonker and A. J. W. M. van Overbeek: Control of a beam of electrons by an intersecting electron beam (Nature London **164**, 276, 1949, Aug. 13).

A ribbon-shaped beam of electrons (100 V), emerging from an oblong, concave cylindrical oxide-coated cathode, is crossed by a second ribbon-shaped beam (16 V). This second beam is influenced by the space charge of the first one. In consequence of this the current  $I_2$  in the second beam can be made to decrease from  $10^{-5}$  A to  $10^{-6}$  A by varying the potential on the control grid of the first beam. If the anode of the second beam is replaced by a secondary emission multiplier, slopes of several amps per volt can be attained at anode currents of the order of  $10^{-2}$  A.

**1885:** C. J. Bakker: Valve noise and transit time (Wireless Eng. **26**, 277, 1949, Aug.).

The writer gives an answer to criticism by Campbell, Francis and James and by Holding (Wireless Eng. **25** (1948), pages 148 and 372 respectively), regarding his 1941 article on valve noise (Physica **8**, p. 23). Further the reader is referred to a letter to the Editor of the Wireless Eng. by F. L. H. M. Stumpers (see these abstracts, No. 1886).

**1886:** F. L. H. M. Stumpers: Measurements of induced grid noise (Wireless Eng. **26**, 277-278, 1949, Aug.).

Short description of an arrangement for measuring induced grid noise after Bakker (Physica **8**, 23, 1941) and report of some measurements in the frequency range 20-100 Mc/s. The measurements are in satisfactory agreement with Bakker's theory.

**1887:** J. J. Went: Relation between the thermal expansion, the Curie temperature and the lattice spacing of homogeneous ternary nickel-iron alloys (Physica The Hague **15**, 703-710, 1949, No. 8/9).

The importance of more experimental data concerning the exchange energy in ferromagnetic alloys is pointed out. The Curie temperature, the expansion anomaly below the Curie temperature and the lattice spacing of 15 only slightly

different homogeneous ternary Ni-Fe alloys with about 50% Fe and 50% Ni are measured. A close relationship exists between the change in Curie temperature and the change in the expansion anomaly between different alloys. The value of this change in the Curie temperature depends upon the position of the third alloying element in the periodic system of elements with respect to the position of Ni in this system. There is no direct relation at all between the change in Curie temperature and the lattice spacing.

**1888:** F. M. Penning and H. J. A. Moubis: On the normal cathode fall in neon (Physica The Hague **15**, 721-732, 1949, No. 8/9).

Applying the same method which had furnished reproducible results for molybdenum, the normal cathode fall  $V_n$  in neon has been determined for a number of other materials (plate cathode, 40 mm pressure). The results are given in table III. The values of  $V_n$  are compared with those of the work function  $\varphi$  and large deviations have been found from the relation  $V_n = C\varphi$  ( $C$  a constant).

**1889:** Th. P. J. Botden and F. A. Kröger: Energy transfer in tungstates and molybdates activated with samarium (Physica The Hague **15**, 746-768, 1949, No. 8/9).

Earth alkali tungstates and molybdates activated with samarium show fluorescence in different bands, which can be correlated with the tungstate group or molybdate group and with trivalent samarium. Upon excitation with  $\lambda$  3560 Å in one of the absorption bands that are characteristic of samarium, only the orange-red samarium fluorescence is emitted. Upon excitation with short-wave ultra-violet in the tungstate or molybdate absorption bands, both types of fluorescence bands are emitted, in a proportion that is dependent on temperature. At low temperatures the efficiencies of both types of fluorescence are constant. At the temperature at which the tungstate or molybdate fluorescence is quenched or at a slightly higher temperature, the intensity of the samarium fluorescence increases with increasing temperature up to a constant value; at a still higher temperature the samarium fluorescence is also quenched.

The increase of the samarium fluorescence is due

to the transfer of energy from tungstate or molybdate groups to samarium ions. The results are discussed and calculations have been made on the basis of the theories of Mott-Seitz and of Möglich-Rompe for the quenching of fluorescence. The results constitute an argument in favour of the model of Mott and Seitz.

**1890:** F. A. Kröger: A proof of the associated-pair theory for sensitized luminophors (Physica The Hague **15**, 801-806, 1949, No. 8/9).

For  $\text{Ca}_3(\text{PO}_4)_2$  and  $\text{Sr}_3(\text{PO}_4)_2$  containing manganese as an activator and tin or cerium as a sensitizer the spectral distribution of the manganese fluorescence, the temperature-dependence of its intensity and the temperature-dependence of its decay have been measured. Analysis of the experimental data leads to figures for the transition probabilities of the fluorescence process and the dissipation process in the manganese centres. Comparison shows that these quantities are markedly different for the centres in products containing tin and cerium as sensitizers. This proves that the sensitizer atoms must be close to the activator atoms, as was assumed in the associated-pair theory of sensitization.

**1891:** F. A. Kröger, J. Th. G. Overbeek, J. Goorissen and J. van den Boomgaard: Bismuth as activator in fluorescent solids (J. Electrochem. Soc. **96**, 132-141, 1949, No. 3).

Trivalent bismuth forms fluorescence centers in its own compounds as well as in other systems in which it is present as an activator. The fluorescent emission consists of various bands lying between the ultraviolet and the red end of the spectrum. The relative intensity of these bands depends upon the nature of the host lattice and the temperature of observation.

Sulfates and phosphates of the alkine earth group show predominantly red fluorescence. The red fluorescence of  $\text{Ca}_2\text{P}_2\text{O}_7\text{-Bi}$  shows the remarkable feature that its temperature-dependence is different for excitation by cathode rays and by X-rays.

**R 122:** W. Nijenhuis: A note on a generalized Van der Pol equation (Philips Res. Rep. **4**, 401-406, 1949, No. 6).

The problem of solving the asymmetrical Van der Pol equation  $\ddot{v} - \varepsilon(1 - 2\beta v + v^2)\dot{v} + v = 0$  is equivalent to that of solving an ordinary Van der Pol equation ( $\beta=0$ ) with a constant right-hand side. For large values of  $\varepsilon$  and  $\beta$  a simple approximation of the limit cycle is found from which it follows that the ratio of the maximum amplitudes in the positive and negative directions cannot exceed 3:1.

**R 123:** J. A. Haringx: On highly compressible helical springs and rubber rods, and their application for vibration-free mountings, VI (Philips Res. Rep. **4**, 407-448, 1949, No. 6).

This paper (see Nos. **R 94**, **R 101**, **R 109**, **R 113** and **R 121**) deals with the actual construction of some vibration-free mountings, in particular a damped dynamic vibration absorber supported by helical springs and an elementary mounting without auxiliary mass supported by rubber rods. Their design is completely based upon the results found in the preceding papers, special attention being drawn to constructions approximately meeting the demands for the separation of the various degrees of freedom. In the introduction it is shown what interfering vibrations may be expected and what additional advantage is yielded by a large logarithmic decrement of the mounting as regards the free vibrations of an undamped instrument placed upon it. The characteristic differences between helical springs and rubber rods when used as resilient elements are mentioned, the latter requiring a much larger mass for the same low resonant frequency of the system. Further, a comprehensive treatment of the various resilient and damping elements is given in order to arrive at a number of formulae and graphs simplifying their design.

**R 124:** R. Loosjes and H. J. Vink: The conduction mechanism in oxide-coated cathodes (Philips Res. Rep. **4**, 449-475, 1949, No. 6).

For the contents of this article see these abstracts, No. **1808\*** and Philips Techn. Rev. **11**, 275-281, 1949/50, No. 9.